Centre of Mass of a System of Particles

Specification: AQA 7367: ME1 · Edexcel 9FM0: FM2 Centres of mass of plane figures (particles)

INDEPENDENT

You do

A · Particles on a line Origin, Σm, Σmx, divide.

1 Particles of mass 2 kg and 6 kg are at $x = 0$ and $x = 4$. Find $\bar{x}$.

$\bar{x} = (2 \times 0 + 6 \times 4) \div 8$ — Σmx ÷ Σm

$= 24 \div 8 = 3$ — evaluate

2 Particles of mass 3, 2 and 5 kg are attached to a light rod AB of length 4 m, at 1 m, 2.5 m and 4 m from A. Find the distance of the centre of mass from A.



$\Sigma mx = 3 + 5 + 20 = 28, \Sigma m = 10$ — origin at A

$\bar{x} = 2.8$ m — Σmx ÷ Σm



picture: G on the rod

3 Particles of mass 1, 4 and 5 kg lie on the x-axis at $x = -3$, $x = 2$ and $x = 1$. Find $\bar{x}$.



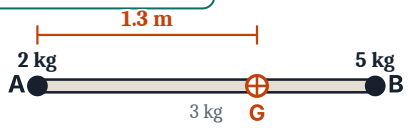
$\Sigma mx = -3 + 8 + 5 = 10$ — keep the minus sign

$\bar{x} = 10 \div 10 = 1$ — Σmx ÷ Σm

4 A uniform rod AB of length 2 m has mass 3 kg. Particles of mass 2 kg and 5 kg are attached at A and B. Find the distance of the centre of mass from A.

Rod: 3 kg at 1 m; $\Sigma mx = 0 + 3 + 10 = 13$ — uniform rod: mass at midpoint

$\bar{x} = 13 \div 10 = 1.3$ m — Σmx ÷ Σm



picture: G on the rod

B · Particles in a plane x and y separately: use a table.

5 Particles of mass 2, 3 and 5 kg are at $(1, 2)$, $(4, 1)$ and $(2, 5)$. Find their centre of mass.

$\Sigma mx = 2 + 12 + 10 = 24, \Sigma my = 4 + 3 + 25 = 32$ — table: mx, my

G is at $(2.4, 3.2)$ — ÷ Σm = 10

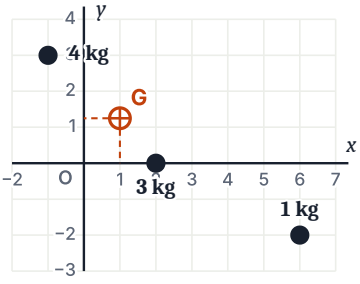


picture: G inside the triangle

6 Particles of mass 4, 1 and 3 kg are at $(-1, 3)$, $(6, -2)$ and $(2, 0)$. Find their centre of mass.

$\Sigma m = 8, \Sigma mx = -4 + 6 + 6 = 8, \Sigma my = 12 - 2 + 0 = 10$ — table: keep the signs

G is at $(1, 1.25)$ — ÷ Σm



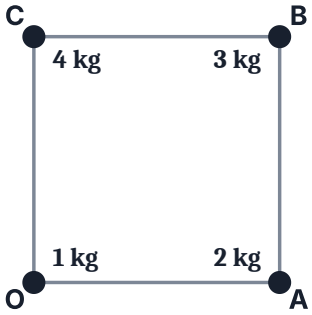
picture: G inside the triangle

7 Particles of mass 2, 3 and 5 kg have position vectors $3\mathbf{i} + \mathbf{j}$, $-\mathbf{i} + 4\mathbf{j}$ and $2\mathbf{i} - 2\mathbf{j}$ metres. Find the position vector of their centre of mass.

$\Sigma m\mathbf{r} = 2(3,1) + 3(-1,4) + 5(2,-2) = (13,4)$ — i and j separately

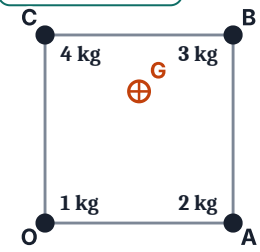
$\bar{\mathbf{r}} = (13,4) \div 10 = 1.3\mathbf{i} + 0.4\mathbf{j}$ — ÷ Σm

8 A light square lamina OABC has side 2 m, with O at the origin, A on the positive x-axis and C on the positive y-axis. Particles of mass 1, 2, 3 and 4 kg are at O, A, B and C. Find the centre of mass.



$\Sigma mx = 2\times2 + 3\times2 = 10$, $\Sigma my = 3\times2 + 4\times2 = 14$ — corners: (0, 0), (2, 0), (2, 2), (0, 2)

G is at $(1, 1.4)$ — ÷ Σm = 10



picture: G on the lamina

C · Reasoning No calculator. Explain in words as well as numbers.

9 Three particles, each of mass m, are at $(0,0)$, $(6,0)$ and $(0,3)$. Show that their centre of mass is the mean of their coordinates, and find it.

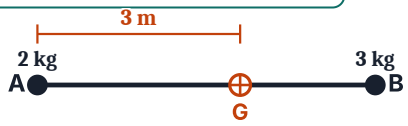
$\bar{x} = (m\times0 + m\times6 + m\times0) \div 3m = 6 \div 3$ — m cancels

$\bar{x} = 2, \bar{y} = 3 \div 3 = 1$ — mean of the coordinates

10 Particles of mass 2 kg at A and 3 kg at B are joined by a light rod of length 5 m. Explain why G lies on AB, nearer to B, and find AG.

Both particles lie on AB, so G does too. It is a weighted mean, so it is nearer the heavier mass at B. — weighted mean

AG = $(2\times0 + 3\times5) \div 5 = 3$ m — origin at A



picture: nearer B

11 Every mass in a system of particles is doubled. What happens to the centre of mass? Explain.

$\bar{x} = \Sigma(2m)x \div \Sigma(2m) = 2\Sigma mx \div 2\Sigma m = \Sigma mx \div \Sigma m$ — factor 2 cancels

G does not move. The same is true for $\bar{y}$. — only ratios matter

D · Unknown masses and positions Write $(\Sigma m)\bar{x} = \Sigma mx$, then solve.

12 Particles of mass 3 kg, 2 kg and m kg are at $x = 0$, $x = 5$ and $x = 10$. Their centre of mass is at $x = 6$. Find m.

$(5 + m)\times6 = 10 + 10m$ — $(\Sigma m)\bar{x} = \Sigma mx$

$30 + 6m = 10 + 10m \Rightarrow m = 5$ — solve

13 Particles of mass 2, 4 and 2 kg are at $(0,0)$, $(a,3)$ and $(6,b)$. Their centre of mass is $(3,2)$. Find a and b.

x: $4a + 12 = 8\times3 \Rightarrow a = 3$ — $(\Sigma m)\bar{x} = \Sigma mx$

y: $12 + 2b = 8\times2 \Rightarrow b = 2$ — $(\Sigma m)\bar{y} = \Sigma my$

$a = 3, b = 2$ — both values

14 Particles of mass m, $2m$ and $3m$ are at $(0, 0)$, $(3a, 0)$ and $(a, 2a)$. Show that their centre of mass is at $(1.5a, a)$.

$\bar{x} = (0 + 6ma + 3ma) \div 6m = 9a \div 6 = 1.5a$ — $\Sigma m = 6m$

$\bar{y} = (3m \times 2a) \div 6m = a$ — m cancels

G is at $(1.5a, a)$ — as coordinates

15 A light rod AB has length 4 m. Particles of mass 2 kg at A, 3 kg at B and 5 kg at C, on AB, have their centre of mass at the midpoint of AB. Find AC.

Let AC = c: $10 \times 2 = 3 \times 4 + 5c$ — $(\Sigma m)\bar{x} = \Sigma mx$

$5c = 8 \Rightarrow c = 1.6$ m — solve



picture: G at the midpoint

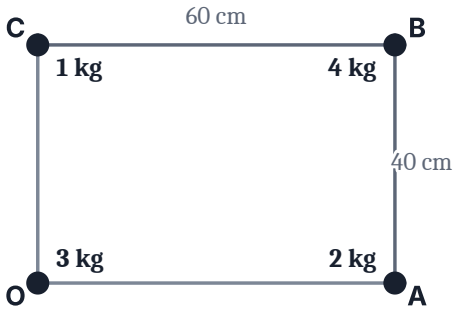
E · Exam-style question Show your method: marks are for working.

16 A light rectangular lamina OABC has O at the origin, OA = 60 cm along the positive x-axis and OC = 40 cm along the positive y-axis. Particles of mass 3, 2, 4 and 1 kg are at O, A, B and C.

(a) Find the centre of mass. [4 marks]

(b) A particle of mass k kg is added at the midpoint of OC. The centre of mass is now on the line $x = 30$. Find k. [2 marks]

(c) Show that $\bar{y}$ does not change. Explain why. [2 marks]



(a) $\Sigma m = 10$, $\Sigma mx = 2 \times 60 + 4 \times 60 = 360$ M1 — table: mx

$\bar{x} = 36$ A1 — ÷ Σm

$\Sigma my = 4 \times 40 + 1 \times 40 = 200$, $\bar{y} = 20$ M1 A1 — table: my

(b) Midpoint of OC is $(0, 20)$: $360 \div (10 + k) = 30$ M1 — $(\Sigma m)\bar{x} = \Sigma mx$

$k = 2$ A1 — solve

(c) $\bar{y} = (200 + 2 \times 20) \div 12 = 240 \div 12 = 20$ M1 — new Σmy ÷ new Σm

Unchanged: the new particle is at the same height as G, $y = 20$. A1 — same height as G