# Rods, Wire Frameworks and Wires of Different Densities

Specification: AQA 7367: ME1, ME2 · Edexcel 9FM0: FM2 Centres of mass of plane figures (frameworks, circular arcs)

INDEPENDENT

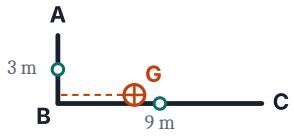
## You do

**A · One wire** Lengths in place of masses: ΣLx ÷ ΣL.

**1** A uniform wire is bent at B into AB = 3 m and BC = 9 m, at right angles. Find the distances of the centre of mass from BC and from BA.

$\Sigma L = 12$, $\Sigma Lx = 9 \times 4.5 = 40.5$, $\Sigma Ly = 3 \times 1.5 = 4.5$ — lengths and midpoints

0.375 m from BC, 3.375 m from BA — ÷ ΣL



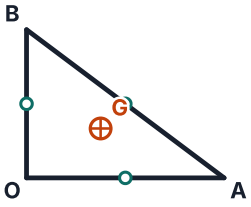

picture: G off the wire

**2** A framework OAB is made from uniform wire, with O at the origin, A at $(16, 0)$ and B at $(0, 12)$. Find its centre of mass.

AB = 20; midpoints $(8, 0)$, $(8, 6)$, $(0, 6)$ — Pythagoras; midpoints

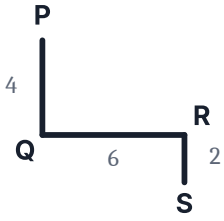
$\Sigma L = 48$, $\Sigma Lx = 128 + 160 = 288$, $\Sigma Ly = 120 + 72 = 192$ — ΣLx, ΣLy

G is at $(6, 4)$ — ÷ ΣL



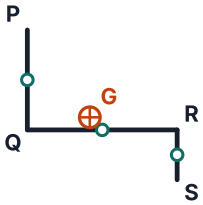

picture: G inside the frame

**3** A uniform wire PQRS is bent into three straight pieces: P $(0, 4)$, Q $(0, 0)$, R $(6, 0)$ and S $(6, -2)$. Find its centre of mass.




$\Sigma L = 12$, $\Sigma Lx = 0 + 18 + 12 = 30$, $\Sigma Ly = 8 + 0 - 2 = 6$ — midpoints (0, 2), (3, 0), (6, −1)

G is at $(2.5, 0.5)$ — ÷ ΣL

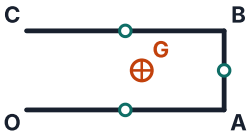



picture: G off the wire

**4** A uniform wire forms three sides of a rectangle: OA = 10 cm along the $x$-axis from the origin O, AB = 4 cm and BC = 10 cm, with C at $(0, 4)$. Side OC is missing. Find the centre of mass.

$\Sigma L = 24$, $\Sigma Lx = 50 + 40 + 50 = 140$ — midpoints (5, 0), (10, 2), (5, 4)

$\bar{x} = 140 \div 24 = 5.83$; $\bar{y} = 2$ by symmetry — ÷ ΣL; symmetry




picture: G nearer AB

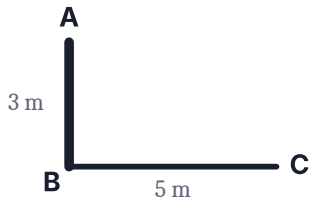
## B · Different densities and particles Masses first: m = λL.

**5** Two uniform rods are joined end to end in a straight line ABC. AB has length 2 m and mass 3 kg; BC has length 4 m and mass 1 kg. Find the distance of the centre of mass from A.

$\Sigma mx = 3 \times 1 + 1 \times 4 = 7, \Sigma m = 4$ — midpoints 1 m and 4 m from A

$\bar{x} = 1.75$ m — ÷ Σm

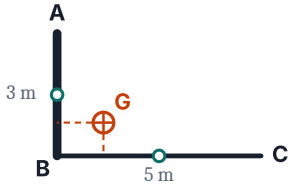
**6** Rods AB and BC are joined at right angles at B. AB has length 3 m and mass 2 kg m$^{-1}$; BC has length 5 m and mass 1 kg m$^{-1}$. Find the distances of the centre of mass from BC and from BA.




Masses: AB 6 kg at $(0, 1.5)$, BC 5 kg at $(2.5, 0)$ — m = λL

$\bar{y} = 9 \div 11 = 0.818, \bar{x} = 12.5 \div 11 = 1.14$ — ÷ Σm = 11

0.818 m from BC, 1.14 m from BA — in context, with units



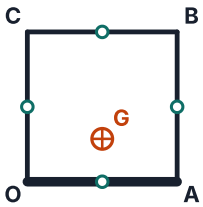

picture: dense AB pulls G

**7** A square framework OABC has side 2 m, with O at the origin, A on the positive $x$-axis and C on the positive $y$-axis. Side OA has mass 4 kg m$^{-1}$; the other three sides have mass 1 kg m$^{-1}$. Find the centre of mass.

Masses: OA 8, AB, BC, CO 2 each; $\bar{x} = 1$ by symmetry — m = λL; symmetry

$\Sigma my = 2 \times 1 + 2 \times 2 + 2 \times 1 = 8, \bar{y} = 8 \div 14 = 0.571$ — ÷ Σm = 14

G is at $(1, 0.571)$ — as coordinates



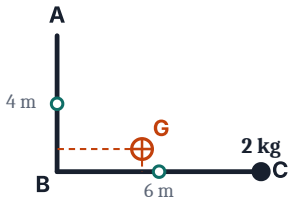

picture: heavy OA pulls G down

**8** A uniform wire of mass 1 kg m$^{-1}$ is bent at B into AB = 4 m and BC = 6 m, at right angles. A particle of mass 2 kg is attached at C. Find the distances of the centre of mass from BC and from BA.

AB 4 kg at $(0, 2)$; BC 6 kg at $(3, 0)$; particle 2 kg at $(6, 0)$ — rods and particle in one table

$\bar{x} = 30 \div 12 = 2.5, \bar{y} = 8 \div 12 = 0.667$ — ÷ Σm = 12

0.667 m from BC, 2.5 m from BA — in context, with units




picture: particle pulls G

## C · Reasoning No calculator. Explain in words.

**9** A square framework is made from uniform wire, with one diagonal added. Explain why its centre of mass is at the centre of the square.

Opposite sides are equal rods with midpoints symmetric about the centre; the diagonal's midpoint is the centre. — pair the rods

Every pair balances about the centre, so G is at the centre. — symmetry

**10** True or false? "A triangular framework of uniform wire and a uniform triangular lamina with the same corners $(0, 0)$, $(6, 0)$, $(0, 8)$ have the same centre of mass." Show your working.

Frame: sides 6, 10, 8; $\bar{x} = (18 + 30) \div 24 = 2$, $\bar{y} = (40 + 32) \div 24 = 3$ — midpoints (3, 0), (3, 4), (0, 4)

Lamina: mean of the corners $(2, 8/3)$. False: $3 \neq 8/3$ — frame ≠ lamina

**11** A semicircular arc of wire and a semicircular lamina have the same radius $r$. The lamina's centre of mass is $4r/3\pi$ from the centre. Which centre of mass is further from the centre? Explain.

Arc: $2r/\pi = 6r/3\pi$, which is more than $4r/3\pi$ — common denominator

The arc is further out: all its mass is on the rim. — mass at the rim

**D · Unknowns and show that** Write $(\Sigma m)\bar{x} = \Sigma mx$, or $\Sigma L\ \bar{x} = \Sigma Lx$, then solve.

**12** A uniform wire is bent at B into AB = $a$ and BC = $2a$, at right angles. Show that its centre of mass is $2a/3$ from BA and $a/6$ from BC.

$\bar{x} = (2a \times a) \div 3a = 2a/3$ — BC: length 2a, midpoint a from B

$\bar{y} = (a \times a/2) \div 3a = a/6$ — AB: midpoint a/2 from B

**13** A uniform wire is bent at B into AB = 4 cm and BC = $L$ cm, at right angles. The centre of mass is 0.5 cm from BC. Find $L$.

$(4 + L) \times 0.5 = 4 \times 2$ — $\Sigma L\ \bar{y} = \Sigma Ly$

$4 + L = 16 \Rightarrow L = 12$ — solve

**14** Rods AB and BC are joined at right angles at B. AB has length 3 m and mass 2 kg m$^{-1}$. BC has length 6 m and mass $\lambda$ kg m$^{-1}$. The centre of mass is 2 m from BA. Find $\lambda$.

$(6 + 6\lambda) \times 2 = 6\lambda \times 3$ — $(\Sigma m)\bar{x} = \Sigma mx$

$12 + 12\lambda = 18\lambda \Rightarrow \lambda = 2$ — solve

**15** A uniform wire forms a semicircular arc of radius $r$ and the diameter joining its ends. Show that the centre of mass is $2r/(\pi + 2)$ from the diameter.

Arc: length $\pi r$ at $2r/\pi$; diameter: $2r$ at 0 — arc result

$\bar{y} = (\pi r \times 2r/\pi) \div (\pi r + 2r) = 2r \div (\pi + 2)$ — $\Sigma Ly \div \Sigma L$; r cancels




picture: G between arc and diameter

**E · Exam-style question** Show your method: marks are for working.

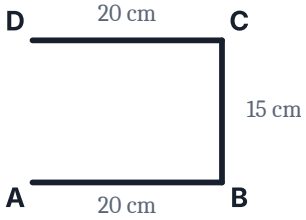
**16** A uniform wire ABCD forms three sides of a rectangle: AB = 20 cm, BC = 15 cm, CD = 20 cm. G is its centre of mass.

(a) Explain why G is 7.5 cm from AB. [1 mark]

(b) Show that G is 140/11 cm from AD. [3 marks]

(c) The wire has mass 0.02 kg per cm. A particle of mass $M$ kg at D moves G to 10 cm from AD. Find $M$. [2 marks]

(d) Find the new distance of G from AB. [2 marks]




**(a)** The wire is symmetric about the line halfway between AB and CD, which is $15 \div 2 = 7.5$ cm from AB, so G is on it. B1 — symmetry

**(b)** Midpoints from AD: AB 10, BC 20, CD 10 M1 — lengths and midpoints

$\bar{x} = (20 \times 10 + 15 \times 20 + 20 \times 10) \div 55 = 700 \div 55$ M1 — $\Sigma Lx \div \Sigma L$

$= 140/11$ cm A1 — simplify

**(c)** Wire mass $55 \times 0.02 = 1.1$ kg: $1.1 \times 140/11 \div (1.1 + M) = 10$ M1 — $(\Sigma m)\bar{x} = \Sigma mx$

$14 = 11 + 10M \Rightarrow M = 0.3$ A1 — solve

**(d)** $(1.1 \times 7.5 + 0.3 \times 15) \div 1.4 = 12.75 \div 1.4$ M1 — D is 15 cm from AB

$= 9.11$ cm A1 — 3 s.f.