Composite Laminas

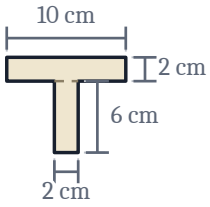
Specification: AQA 7367: ME2 · Edexcel 9FM0: FM2 Centres of mass of plane figures (composite laminas, standard results)

INDEPENDENT

You do

A · Rectangles, L-shapes and holes One row per part; a hole has a negative area.

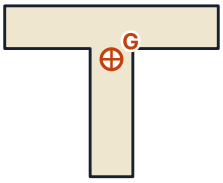
1 A uniform T-shaped lamina is made of a bar 10 cm by 2 cm on top of a stem 2 cm wide and 6 cm tall, as shown. Find the distance of its centre of mass from the bottom of the stem.



Symmetric: G is on the centre line of the stem. — symmetry first

Bar 20 at 7; stem 12 at 3: $\bar{y} = (140 + 36) \div 32$ — $\Sigma Ay \div \Sigma A$

= 5.5 cm — in context, with units



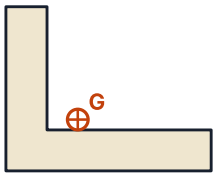
picture: G in the bar

2 A uniform lamina has corners $(0, 0)$, $(10, 0)$, $(10, 2)$, $(2, 2)$, $(2, 8)$ and $(0, 8)$. Find its centre of mass.

Bottom 20 at $(5, 1)$; upright $2 \times 6 = 12$ at $(1, 5)$ — split into rectangles

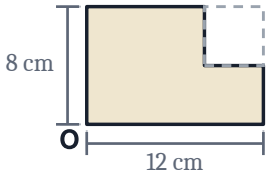
$\Sigma A = 32$, $\Sigma Ax = 112$, $\Sigma Ay = 80$ — table totals

G is at $(3.5, 2.5)$ — $\div \Sigma A$



picture: G off the lamina

3 A square of side 4 cm is cut from the top right corner of a uniform rectangle 12 cm by 8 cm. Find the centre of mass of what is left, taking O at the bottom left corner.



Rectangle 96 at $(6, 4)$; removed -16 at $(10, 6)$ — removed: negative area

$\bar{x} = (576 - 160) \div 80 = 5.2$, $\bar{y} = (384 - 96) \div 80 = 3.6$ — $\div \Sigma A = 80$

G is at $(5.2, 3.6)$ — as coordinates



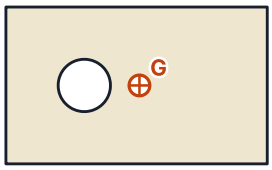
picture: G away from the cut

4 A uniform rectangle $0 \le x \le 10$, $0 \le y \le 6$ (in cm) has a circular hole of radius 1 cm centred at $(3, 3)$. Find the centre of mass.

$\bar{y} = 3$ by symmetry; areas 60 and $-\pi$ — symmetry; hole negative

$\bar{x} = (300 - 3\pi) \div (60 - \pi) = 5.11$ — $\Sigma Ax \div \Sigma A$

G is at $(5.11, 3)$ — as coordinates



picture: G away from the hole

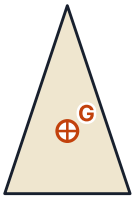
B · Standard results

Triangle: mean of the vertices. Semicircle: 4r/3π. Sector: 2r sin α ÷ 3α.

5 A uniform triangular lamina has vertices $(1, 2)$, $(7, 2)$ and $(4, 11)$. Find its centre of mass.

$((1 + 7 + 4) \div 3, (2 + 2 + 11) \div 3)$ — mean of the vertices

$= (4, 5)$ — as coordinates



picture: G a third of the way up

6 A uniform semicircular lamina has radius 9 cm. Find the distance of its centre of mass from its diameter.

$4r/3\pi = 36 \div 3\pi = 12/\pi$ — semicircle result

$= 3.82$ cm — 3 s.f.

7 A uniform sector has radius 12 cm and angle 90° at its centre O. Find the distance of its centre of mass from O.

$\alpha = \pi/4$: $2 \times 12 \times \sin(\pi/4) \div (3\pi/4) = 16\sqrt{2}/\pi$ — 2r sin α ÷ 3α

$= 7.20$ cm — 3 s.f.

8 A uniform lamina is a square OABC of side 6 cm with a right-angled triangle ABD attached along AB, where D is $(10, 0)$. O is the origin, A is $(6, 0)$ and B is $(6, 6)$. Find the centre of mass.

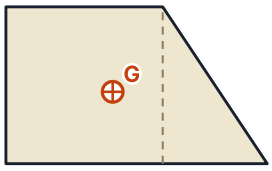
Square 36 at $(3, 3)$; triangle 12 at $((6 + 6 + 10) \div 3, 6 \div 3) = (22/3, 2)$

triangle: mean of the vertices

$\bar{x} = (108 + 88) \div 48 = 4.08$, $\bar{y} = (108 + 24) \div 48 = 2.75$

÷ ΣA = 48

G is at $(4.08, 2.75)$ — as coordinates



picture: G towards the triangle

C · Reasoning

No calculator. Explain in words.

9 True or false? "The centre of mass of a uniform lamina always lies on the lamina." Give a counter-example.

False. A uniform square with a square hole cut from its centre has G at the centre of the hole, by symmetry.

counter-example

G is a balance point; it need not be on the material.

G need not be on the body

10 Show that the sector result $2r \sin \alpha \div 3\alpha$ gives $4r/3\pi$ for a semicircular lamina.

A semicircle is a sector with $2\alpha = \pi$, so $\alpha = \pi/2$ and $\sin \alpha = 1$

2α = π

$2r \times 1 \div (3\pi/2) = 4r/3\pi$ — simplify

11 A uniform strip lies along $0 \le x \le 8$. It is folded in half along $x = 4$, so the right half lies on the left half. Where is G now? Explain without a table.

After folding, both halves lie on $0 \le x \le 4$, each with its centre at $x = 2$

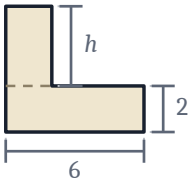
the folded half is reflected in x = 4

Two equal masses at the same point: $\bar{x} = 2$

equal masses, same centre

D · Unknowns, densities and show that Write $(\Sigma A)\bar{x} = \Sigma Ax$, or $(\Sigma m)\bar{x} = \Sigma mx$, then solve.

12 A uniform L-shaped lamina is made of a rectangle $0 \le x \le 6$, $0 \le y \le 2$ and a rectangle $0 \le x \le 2$, $2 \le y \le 2 + h$. Its centre of mass is at $\bar{y} = 2$. Show that $h = 2\sqrt{3}$.

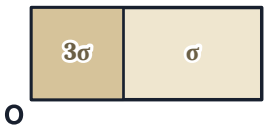


Areas 12 at $y = 1$ and $2h$ at $y = 2 + h/2$ — areas and centres

$(12 + 2h) \times 2 = 12 + 2h(2 + h/2)$ — $(\Sigma A)\bar{y} = \Sigma Ay$

$24 + 4h = 12 + 4h + h^2 \Rightarrow h^2 = 12 \Rightarrow h = 2\sqrt{3}$ — solve: h > 0

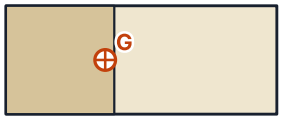
13 A lamina is made of two rectangles joined along the line $x = 4$: the rectangle $0 \le x \le 4$, $0 \le y \le 4$ has mass 3σ per unit area and the rectangle $4 \le x \le 10$, $0 \le y \le 4$ has mass σ per unit area. Find the centre of mass.



Masses: $3\sigma \times 16 = 48\sigma$ at $x = 2$; $\sigma \times 24 = 24\sigma$ at $x = 7$ — masses first: σA

$\bar{x} = (96\sigma + 168\sigma) \div 72\sigma = 11/3 = 3.67$, $\bar{y} = 2$ — Σmx ÷ Σm; symmetry

G is at $(3.67, 2)$ — as coordinates



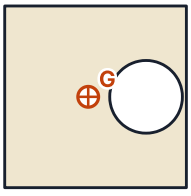
picture: G towards the denser part

14 A uniform square lamina OABC of side 10 cm, with O at the origin, OA along the x-axis and OC along the y-axis, has a circular hole of radius 2 cm centred at $(c, 5)$. The centre of mass is at $\bar{x} = 4.6$. Find c.

$(100 - 4\pi) \times 4.6 = 100 \times 5 - 4\pi \times c$ — $(\Sigma A)\bar{x} = \Sigma Ax$

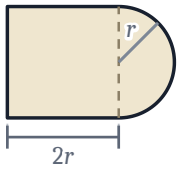
$4\pi c = 40 + 18.4\pi \Rightarrow c = 10/\pi + 4.6$ — solve for c

$c = 7.78$ — 3 s.f.



picture: hole at c = 7.78

15 A uniform lamina is a square of side $2r$ with a semicircle of radius r attached along one side. Show that its centre of mass is a distance $\dfrac{20r}{3(\pi + 8)}$ from the join, inside the square.



Square: $4r^2$ at $-r$; semicircle: $\pi r^2/2$ at $+4r/3\pi$ (the join as $x = 0$) — areas; signs either side

$\Sigma Ax = -4r^3 + 2r^3/3 = -10r^3/3$; $\Sigma A = r^2(8 + \pi)/2$ — moments about the join

$\bar{x} = \dfrac{-10r^3/3}{r^2(8 + \pi)/2} = -\dfrac{20r}{3(\pi + 8)}$: inside the square

negative: square side

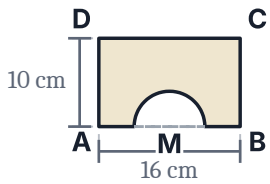
E · Exam-style question Show your method: marks are for working.

16 A uniform lamina is a rectangle ABCD, AB = 16 cm, AD = 10 cm, with a semicircle of radius 4 cm removed. Its diameter lies along AB and its centre is the midpoint M of AB. G is the centre of mass.

(a) Explain why G is 8 cm from AD. [1 mark]

(b) Find the distance of G from AB. [4 marks]

(c) The lamina has mass 3 kg. A particle of mass k kg is attached at D. G is now 7 cm from AD. Find k. [3 marks]



(a) The lamina is symmetric about the perpendicular to AB through M, which is 8 cm from AD. **B1** — symmetry

(b) Rectangle 160 at 5; semicircle -8π at $16/(3\pi)$ from AB **B1** — $4r/3\pi$; removed: negative

$\Sigma Ay = 800 - 128/3 = 2272/3$, $\bar{y} = (2272/3) \div (160 - 8\pi)$ **M1 M1** — $\Sigma Ay \div \Sigma A$

$= 5.62$ cm **A1** — 3 s.f.

(c) Moments about AD: $(3 + k) \times 7 = 3 \times 8 + k \times 0$ **M1 A1** — D is on AD

$21 + 7k = 24 \Rightarrow k = 3/7$ **A1** — solve