Composite Laminas

Specification: AQA 7367: ME2 · Edexcel 9FM0: FM2 Centres of mass of plane figures (composite laminas, standard results)

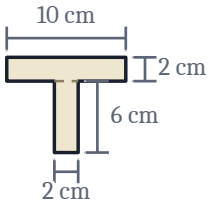
INDEPENDENT

You do

A · Rectangles, L-shapes and holes

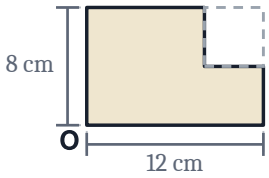
One row per part; a hole has a negative area.

1 A uniform T-shaped lamina is made of a bar 10 cm by 2 cm on top of a stem 2 cm wide and 6 cm tall, as shown. Find the distance of its centre of mass from the bottom of the stem.



2 A uniform lamina has corners $(0, 0)$, $(10, 0)$, $(10, 2)$, $(2, 2)$, $(2, 8)$ and $(0, 8)$. Find its centre of mass.

3 A square of side 4 cm is cut from the top right corner of a uniform rectangle 12 cm by 8 cm. Find the centre of mass of what is left, taking O at the bottom left corner.



4 A uniform rectangle $0 \le x \le 10$, $0 \le y \le 6$ (in cm) has a circular hole of radius 1 cm centred at $(3, 3)$. Find the centre of mass.

B · Standard results

Triangle: mean of the vertices. Semicircle: $4r/3\pi$. Sector: $2r \sin \alpha \div 3\alpha$.

5 A uniform triangular lamina has vertices $(1, 2)$, $(7, 2)$ and $(4, 11)$. Find its centre of mass.

6 A uniform semicircular lamina has radius 9 cm. Find the distance of its centre of mass from its diameter.

7 A uniform sector has radius 12 cm and angle 90° at its centre O. Find the distance of its centre of mass from O.

8 A uniform lamina is a square OABC of side 6 cm with a right-angled triangle ABD attached along AB, where D is $(10, 0)$. O is the origin, A is $(6, 0)$ and B is $(6, 6)$. Find the centre of mass.

C · Reasoning

No calculator. Explain in words.

9 True or false? "The centre of mass of a uniform lamina always lies on the lamina." Give a counter-example.

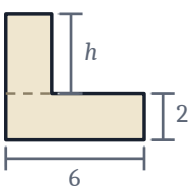
10 Show that the sector result $2r \sin \alpha \div 3\alpha$ gives $4r/3\pi$ for a semicircular lamina.

11 A uniform strip lies along $0 \le x \le 8$. It is folded in half along $x = 4$, so the right half lies on the left half. Where is G now? Explain without a table.

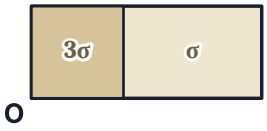
D · Unknowns, densities and show that

Write $(\Sigma A)\bar{x} = \Sigma Ax$, or $(\Sigma m)\bar{x} = \Sigma mx$, then solve.

12 A uniform L-shaped lamina is made of a rectangle $0 \le x \le 6$, $0 \le y \le 2$ and a rectangle $0 \le x \le 2$, $2 \le y \le 2 + h$. Its centre of mass is at $\bar{y} = 2$. Show that $h = 2\sqrt{3}$.

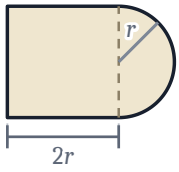


13 A lamina is made of two rectangles joined along the line $x = 4$: the rectangle $0 \le x \le 4$, $0 \le y \le 4$ has mass 3σ per unit area and the rectangle $4 \le x \le 10$, $0 \le y \le 4$ has mass σ per unit area. Find the centre of mass.



14 A uniform square lamina OABC of side 10 cm, with O at the origin, OA along the x-axis and OC along the y-axis, has a circular hole of radius 2 cm centred at $(c, 5)$. The centre of mass is at $\bar{x} = 4.6$. Find c.

15 A uniform lamina is a square of side $2r$ with a semicircle of radius r attached along one side. Show that its centre of mass is a distance $\dfrac{20r}{3(\pi + 8)}$ from the join, inside the square.



E · Exam-style question Show your method: marks are for working.

16 A uniform lamina is a rectangle ABCD, AB = 16 cm, AD = 10 cm, with a semicircle of radius 4 cm removed. Its diameter lies along AB and its centre is the midpoint M of AB. G is the centre of mass.

(a) Explain why G is 8 cm from AD. [1 mark]

(b) Find the distance of G from AB. [4 marks]

(c) The lamina has mass 3 kg. A particle of mass k kg is attached at D. G is now 7 cm from AD. Find k. [3 marks]

