Freely Suspended Laminas and Frameworks

Specification: AQA 7367: ME2 (bodies freely suspended) · Edexcel 9FM0: FM2 Centres of mass of plane figures (hanging)

INDEPENDENT

You do

A · The angle from a known G Distances from the point of suspension: tan θ = across ÷ along.

1 A uniform lamina has a straight edge OA. Its centre of mass is 3 cm from O measured along OA and 4 cm from OA. The lamina is freely suspended from O. Find the angle between OA and the vertical.

$\tan\theta = 4/3$ — tan θ = across ÷ along

$\theta = 53.1°$ — calculator in degrees

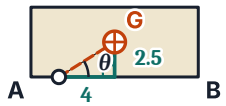
2 A uniform rectangular lamina ABCD has AB = 10 cm and BC = 4 cm. It is freely suspended from A. Find the angle between AB and the vertical.

G is at the centre: along AB 5, across 2 — symmetry: G at the centre

$\theta = \tan^{-1}(2/5) = 21.8°$ — tan θ = across ÷ along

3 A uniform rectangular lamina ABCD has AB = 12 cm and BC = 5 cm. It is freely suspended from the point P on AB with AP = 2 cm. Find the angle between AB and the vertical.

From P: along PB 6 − 2 = 4, across 2.5 — distances from P

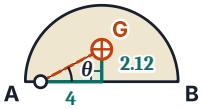


picture: triangle from P

$\theta = \tan^{-1}(2.5/4) = 32.0°$ — tan θ = across ÷ along

4 A uniform semicircular lamina has radius 5 cm, diameter AB and centre O. It is freely suspended from the point P on AB with AP = 1 cm. Find the angle between AB and the vertical.

G is $4r/3\pi = 20/(3\pi) = 2.122$ from the centre; from P: along 5 − 1 = 4, across 2.122 — 4r/3π; distances from P



picture: triangle from P

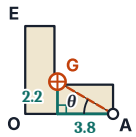
$\theta = \tan^{-1}(2.122/4) = 27.9°$ — tan θ = across ÷ along

B · Find G first Lessons 1–3 for G, then the triangle from the point of suspension.

5 A uniform L-shaped lamina has corners O $(0, 0)$, A $(6, 0)$, B $(6, 2)$, C $(2, 2)$, D $(2, 6)$ and E $(0, 6)$. It is freely suspended from A. Find the angle between AO and the vertical.

Bottom 12 at $(3, 1)$, upright 8 at $(1, 4)$: G $(2.2, 2.2)$ — G by the table (L3)

From A: along AO 6 − 2.2 = 3.8, across 2.2 — distances from A



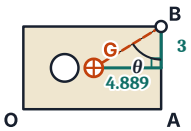
picture: triangle from A

$\theta = \tan^{-1}(2.2/3.8) = 30.1°$ — tan θ = across ÷ along

6 A uniform rectangular lamina OABC, with O $(0, 0)$, A $(10, 0)$, B $(10, 6)$ and C $(0, 6)$ (in cm), has a circular hole of radius 1 cm centred at $(3, 3)$. It is freely suspended from B. Find the angle between BA and the vertical.

$\bar{x} = (300 - 3\pi) \div (60 - \pi) = 5.111$, $\bar{y} = 3$ — hole: negative area

From B: along BA 6 − 3 = 3, across 10 − 5.111 = 4.889 — distances from B



picture: triangle from B

$\theta = \tan^{-1}(4.889/3) = 58.5°$ — tan θ = across ÷ along

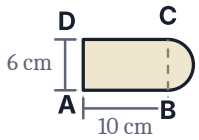
7 A uniform triangular lamina has vertices O $(0,0)$, A $(9,0)$ and B $(6,6)$. It is freely suspended from A. Find the angle between AO and the vertical.

G = mean of the vertices = $(5,2)$ — mean of the vertices

From A: along AO 4, across 2: $\theta = \tan^{-1}(2/4) = 26.6°$

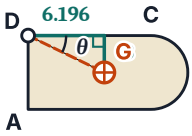
tan θ = across ÷ along

8 A uniform lamina is a rectangle ABCD, AB = 10 cm, BC = 6 cm, with a semicircle on BC outside the rectangle (Lesson 3: G is 6.196 cm from AD). It is freely suspended from D. Find the angle between DC and the vertical.



By symmetry G is 3 cm from AB, so 3 cm from DC — symmetry

From D: along DC 6.196, across 3 — distances from D



picture: triangle from D

$\theta = \tan^{-1}(3/6.196) = 25.8°$

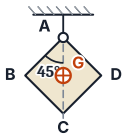
tan θ = across ÷ along

C · Reasoning No calculator. Explain in words.

9 A uniform square lamina is freely suspended from one corner. Explain why the diagonal through that corner hangs vertically, and write down the angle that each edge through that corner makes with the vertical.

The square is symmetric about that diagonal, so G (the centre) lies on it. G hangs below the corner, so the diagonal is vertical.

G on the line of symmetry



picture: diagonal vertical

A diagonal of a square makes 45° with each side: each edge makes 45° with the vertical.

diagonal bisects the right angle

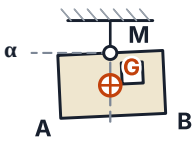
10 A uniform rectangular lamina is freely suspended from the midpoint of one edge.
(a) Explain why that edge hangs horizontally.
(b) A hole is cut in the lamina, away from its line of symmetry. Does the edge still hang horizontally? Explain.

(a) The perpendicular to the edge through its midpoint is a line of symmetry, so G is on it. G hangs below the midpoint, so that line is vertical and the edge is horizontal.

symmetry: G below the midpoint

(b) No. G moves away from the hole, off the line of symmetry, so G is no longer vertically below the midpoint: the edge tilts, with the end further from the hole lower.

G moves away from the hole



picture: it tilts

11 A uniform semicircular lamina is freely suspended from one end of its diameter. Show that the angle between the diameter and the vertical is the same for every radius.

Along the diameter from the end: r; across: $4r/3\pi$ — distances from the end

$\tan\theta = (4r/3\pi) \div r = 4/(3\pi)$: r cancels, so θ is the same for every radius.

r cancels

D · Unknowns, particles and show that Write $\bar{x}$ and $\bar{y}$ in terms of the unknown, use the condition, solve.

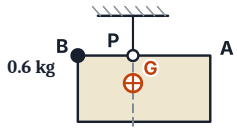
12 A uniform rectangular lamina ABCD has AB = 12 cm, AD = 6 cm and mass 3 kg. A particle of mass m kg is attached at B. The system is freely suspended from the point P on AB with AP = 7 cm, and AB hangs horizontally. Find m.

AB horizontal: G is on the perpendicular to AB through P, so $\bar{x} = 7$ (A at the origin)

horizontal: G below P

$(3 \times 6 + 12m) \div (3 + m) = 7$ (Σm) $\bar{x}$ = Σmx

$18 + 12m = 21 + 7m \Rightarrow m = 0.6$ solve



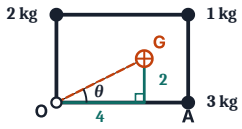
picture: AB horizontal

13 Particles of mass 3 kg, 2 kg and m kg are fixed to a light rectangular frame OABC at A $(6, 0)$, C $(0, 4)$ and B $(6, 4)$ (in m). The frame is freely suspended from O, and OA makes an angle θ with the vertical, where $\tan\theta = ½$. Find m.

$\bar{x} = (18 + 6m) \div (5 + m)$, $\bar{y} = (8 + 4m) \div (5 + m)$ Σmx ÷ Σm (L1)

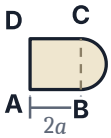
$\tan\theta = \bar{y} \div \bar{x} = (8 + 4m) \div (18 + 6m) = ½$ tan θ = across ÷ along

$16 + 8m = 18 + 6m \Rightarrow m = 1$ (OA at $\tan^{-1} ½ = 26.6°$) solve



picture: m = 1 check

14 A uniform lamina is a square ABCD of side $2a$ with a semicircle of radius a attached along BC, outside the square. Its centre of mass is $\dfrac{20a}{3(\pi + 8)}$ from BC, inside the square (Lesson 3). The lamina is freely suspended from A. Show that AB makes an angle θ with the vertical, where $\tan\theta = \dfrac{3(\pi + 8)}{2(3\pi + 14)}$.



Across AB: a (symmetry). Along AB from A: $2a - 20a/(3(\pi + 8))$

distances from A

$= (6a(\pi + 8) - 20a) \div 3(\pi + 8) = 2a(3\pi + 14) \div 3(\pi + 8)$ common denominator

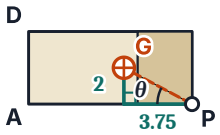
$$\tan\theta = a \div \frac{2a(3\pi + 14)}{3(\pi + 8)} = \frac{3(\pi + 8)}{2(3\pi + 14)}$$

tan θ = across ÷ along

15 A uniform rectangular lamina ABCD has AB = 12 cm and BC = 4 cm. It is folded along the line parallel to BC and 3 cm from BC, so that the folded part lies flat on the rest. (Lesson 3: G is now 5.25 cm from AD.) The folded lamina is freely suspended from the end of the fold on AB. Find the angle between AB and the vertical.

The fold meets AB 9 cm from A. From there: along AB $9 - 5.25 = 3.75$, across 2

distances from the fold



picture: folded lamina

$\theta = \tan^{-1}(2/3.75) = 28.1°$ tan θ = across ÷ along

E · Exam-style question Show your method: marks are for working.

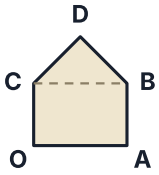
16 A uniform lamina is a rectangle OABC, O $(0, 0)$, A $(12, 0)$, B $(12, 8)$, C $(0, 8)$ in cm, with a triangle CBD on top, D $(6, 14)$.

(a) Explain why its centre of mass G is 6 cm from OC. [1 mark]

(b) Show that G is $62/11$ cm from OA. [3 marks]

(c) It is freely suspended from A. Find the angle between AB and the vertical. [3 marks]

(d) It hangs from D instead. Which line hangs vertically? Explain. [1 mark]



(a) The lamina is symmetric about $x = 6$, so G is on it: 6 cm from OC. **B1** — symmetry

(b) Rectangle 96 at $y = 4$; triangle $\frac{1}{2} \times 12 \times 6 = 36$ at $y = 8 + \frac{1}{3} \times 6 = 10$ **B1** — areas and centres

$\Sigma Ay = 384 + 360 = 744$, $\bar{y} = 744 \div 132 = 62/11$ **M1 A1** — ΣAy ÷ ΣA

(c) From A: along AB $62/11$, across 6 **M1** — distances from A

$\tan\theta = 6 \div (62/11) = 33/31$, $\theta = 46.8°$ **M1 A1** — tan θ; degrees

(d) D and G are on the line of symmetry $x = 6$: it hangs vertically. **B1** — G below D