Freely Suspended Laminas and Frameworks

Specification: AQA 7367: ME2 (bodies freely suspended) · Edexcel 9FM0: FM2 Centres of mass of plane figures (hanging)

INDEPENDENT **You do**

A · The angle from a known G Distances from the point of suspension: tan θ = across ÷ along.

1 A uniform lamina has a straight edge OA. Its centre of mass is 3 cm from O measured along OA and 4 cm from OA. The lamina is freely suspended from O. Find the angle between OA and the vertical.

2 A uniform rectangular lamina ABCD has AB = 10 cm and BC = 4 cm. It is freely suspended from A. Find the angle between AB and the vertical.

3 A uniform rectangular lamina ABCD has AB = 12 cm and BC = 5 cm. It is freely suspended from the point P on AB with AP = 2 cm. Find the angle between AB and the vertical.

4 A uniform semicircular lamina has radius 5 cm, diameter AB and centre O. It is freely suspended from the point P on AB with AP = 1 cm. Find the angle between AB and the vertical.

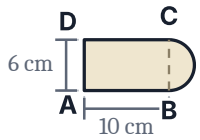
B · Find G first Lessons 1–3 for G, then the triangle from the point of suspension.

5 A uniform L-shaped lamina has corners O $(0, 0)$, A $(6, 0)$, B $(6, 2)$, C $(2, 2)$, D $(2, 6)$ and E $(0, 6)$. It is freely suspended from A. Find the angle between AO and the vertical.

6 A uniform rectangular lamina OABC, with O $(0, 0)$, A $(10, 0)$, B $(10, 6)$ and C $(0, 6)$ (in cm), has a circular hole of radius 1 cm centred at $(3, 3)$. It is freely suspended from B. Find the angle between BA and the vertical.

7 A uniform triangular lamina has vertices O $(0, 0)$, A $(9, 0)$ and B $(6, 6)$. It is freely suspended from A. Find the angle between AO and the vertical.

8 A uniform lamina is a rectangle ABCD, AB = 10 cm, BC = 6 cm, with a semicircle on BC outside the rectangle (Lesson 3: G is 6.196 cm from AD). It is freely suspended from D. Find the angle between DC and the vertical.



C · Reasoning No calculator. Explain in words.

9 A uniform square lamina is freely suspended from one corner. Explain why the diagonal through that corner hangs vertically, and write down the angle that each edge through that corner makes with the vertical.

10 A uniform rectangular lamina is freely suspended from the midpoint of one edge.
(a) Explain why that edge hangs horizontally.
(b) A hole is cut in the lamina, away from its line of symmetry. Does the edge still hang horizontally? Explain.

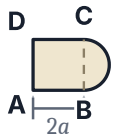
11 A uniform semicircular lamina is freely suspended from one end of its diameter. Show that the angle between the diameter and the vertical is the same for every radius.

D · Unknowns, particles and show that Write $\bar{x}$ and $\bar{y}$ in terms of the unknown, use the condition, solve.

12 A uniform rectangular lamina ABCD has AB = 12 cm, AD = 6 cm and mass 3 kg. A particle of mass m kg is attached at B. The system is freely suspended from the point P on AB with AP = 7 cm, and AB hangs horizontally. Find m.

13 Particles of mass 3 kg, 2 kg and m kg are fixed to a light rectangular frame OABC at A $(6, 0)$, C $(0, 4)$ and B $(6, 4)$ (in m). The frame is freely suspended from O, and OA makes an angle θ with the vertical, where $\tan\theta = \frac{1}{2}$. Find m.

14 A uniform lamina is a square ABCD of side $2a$ with a semicircle of radius a attached along BC, outside the square. Its centre of mass is $\dfrac{20a}{3(\pi+8)}$ from BC, inside the square (Lesson 3). The lamina is freely suspended from A. Show that AB makes an angle θ with the vertical, where $\tan\theta = \dfrac{3(\pi+8)}{2(3\pi+14)}$.



15 A uniform rectangular lamina ABCD has AB = 12 cm and BC = 4 cm. It is folded along the line parallel to BC and 3 cm from BC, so that the folded part lies flat on the rest. (Lesson 3: G is now 5.25 cm from AD.) The folded lamina is freely suspended from the end of the fold on AB. Find the angle between AB and the vertical.

E · Exam-style question Show your method: marks are for working.

16 A uniform lamina is a rectangle OABC, O $(0,0)$, A $(12,0)$, B $(12,8)$, C $(0,8)$ in cm, with a triangle CBD on top, D $(6,14)$.

(a) Explain why its centre of mass G is 6 cm from OC. [1 mark]

(b) Show that G is $62/11$ cm from OA. [3 marks]

(c) It is freely suspended from A. Find the angle between AB and the vertical. [3 marks]

(d) It hangs from D instead. Which line hangs vertically? Explain. [1 mark]

