Unit Review

Specification: AQA 7367: ME1, ME2 · Edexcel 9FM0: FM2 Centres of mass of plane figures, Further centres of mass

INDEPENDENT

Mixed practice

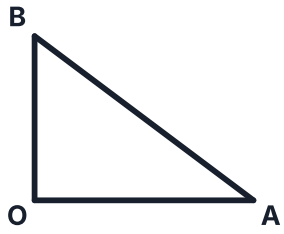
A · Fluency One or two steps each. Which tool?

1 Particles of mass 4 kg, 1 kg and 5 kg lie on the x-axis at $x = -2$, $x = 3$ and $x = 4$. Find $\bar{x}$.

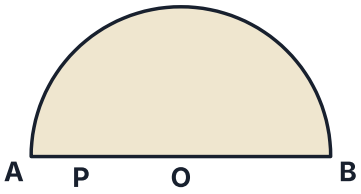
2 A uniform triangular lamina has vertices $(0, 0)$, $(9, 0)$ and $(3, 6)$. Find its centre of mass.

3 (a) A uniform solid cone has height 20 cm.
(b) A uniform conical shell, open at its base, has height 18 cm.
For each, find the distance of the centre of mass from the vertex.

4 A uniform wire is bent into a triangle OAB with $O(0, 0)$, $A(4, 0)$ and $B(0, 3)$. Find the centre of mass of the wire framework.



5 A uniform semicircular lamina has diameter AB = 12 cm and centre O. It is freely suspended from the point P on AB with AP = 2 cm. Find the angle between AB and the vertical.



6 Particles of mass 3 kg and m kg are on the x-axis at $x = 1$ and $x = 6$. Their centre of mass is at $x = 4$. Find m.

B · Multi-step Several tools in one question. Sketch first.

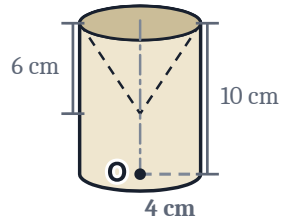
7 A uniform lamina is made of the rectangle OABC, with $O(0, 0)$, $A(8, 0)$, $B(8, 4)$ and $C(0, 4)$, and a semicircle with diameter AB on the outside of the rectangle.
(a) Find the centre of mass of the lamina.
(b) The lamina is freely suspended from O. Find the angle between OA and the vertical.



8 A uniform wire of length 16 cm is bent into three sides AB, BC and CD of a rectangle, with AB = CD = 4 cm and BC = 8 cm.
(a) Find the distance of the centre of mass from BC.
(b) It hangs freely from A. Find the angle between AB and the vertical.

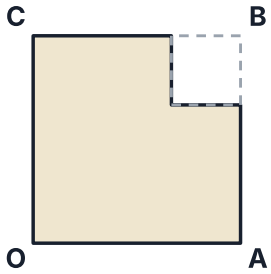


9 A cone of base radius 4 cm and height 6 cm is drilled out of a uniform solid cylinder of radius 4 cm and height 10 cm. The base of the cone is the top face of the cylinder and they have the same axis.
(a) Find the distance of the centre of mass of the remaining solid from the bottom face.
(b) The solid is freely suspended from a point on the rim of its bottom face. Find the angle between its axis and the vertical.



10 A light rectangular lamina OABC has $O(0,0)$, $A(6,0)$, $B(6,4)$ and $C(0,4)$. Particles of mass 2 kg, 3 kg, 1 kg and 4 kg are attached at O, A, B and C.
(a) Find the centre of mass of the system.
(b) An extra particle of mass m kg is attached at B, so that the centre of mass lies on the line $x = 3$. Find m.

11 A uniform square lamina OABC has $O(0,0)$, $A(6,0)$, $B(6,6)$ and $C(0,6)$. The square $4 \le x \le 6$, $4 \le y \le 6$ is cut away.
(a) Find the centre of mass of the remaining lamina.
(b) It is freely suspended from O. Without further calculation, find the angle between OA and the vertical.



C · Reasoning No calculator. Explain in words.

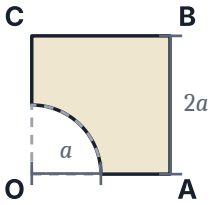
12 Ana says: "A uniform triangular wire framework and a uniform triangular lamina of the same shape always have the same centre of mass." Test her claim with the triangle $O(0,0)$, $A(6,0)$, $B(0,8)$.

13 Each of these is uniform with radius 8 cm: a semicircular arc of wire, a semicircular lamina, a solid hemisphere and a hemispherical shell. Put them in order of the distance of their centre of mass from the centre O, largest first. Use $3 < \pi < 3.2$.

14 A uniform square lamina hangs freely from one corner, so that a diagonal is vertical. A particle is then attached to the lowest corner. Does the position in which the lamina hangs change? Explain.

D · Show that Algebra in the table. Leave π in.

15 A uniform square lamina OABC has side $2a$, with O at the origin, OA along the x-axis and OC along the y-axis. A quarter-circle of radius a and centre O is cut away. Show that the centre of mass of the remaining lamina is at a distance $\dfrac{44a}{3(16-\pi)}$ from OC.



16 A toy is made of a uniform solid hemisphere of radius r and density 2ρ, joined at its plane face (centre O) to a uniform solid cone of base radius r, height h and density ρ. Taking x upwards from O along the axis, show that $\bar{x} = \dfrac{h^2 - 6r^2}{4(h+4r)}$. Hence find the condition on h for the toy to return upright when it rests on its curved surface.

E · Exam-style questions Show your method: marks are for working.

17 *AQA-style*. Lengths are in cm. A uniform lamina is made from the rectangle OABC, where $O(0,0)$, $A(12,0)$, $B(12,6)$ and $C(0,6)$, by cutting away a semicircle of radius 2 cm whose diameter lies along OC, with its centre at $(0,3)$.

(a) Show that the centre of mass of the lamina is at a distance $\dfrac{640}{3(36-\pi)}$ cm from OC. [4 marks]

(b) The lamina is freely suspended from B. Find the angle between BA and the vertical. [2 marks]

(c) The lamina has mass M. A particle of mass kM is attached at C. When the loaded lamina is freely suspended from O, OA makes an angle of $45°$ with the vertical. Find k. [2 marks]

18 *Edexcel-style*. A uniform solid cylinder has radius 3 cm and height 8 cm. A cone of base radius 3 cm and height 4 cm is removed from one end: the base of the cone is that end face and they have the same axis. O is the centre of the other end face.

(a) Show that the centre of mass of the remaining solid is 3.4 cm from O. [4 marks]

(b) The solid is freely suspended from a point A on the rim of the face containing O. Find the angle between the axis and the vertical. [2 marks]

(c) The solid has mass M. A particle of mass kM is attached at O. When the loaded solid is freely suspended from A, its axis makes an angle of $45°$ with the vertical. Find k. [2 marks]