Laminas by Integration

Specification: AQA 7367: ME3 (lamina by integration) · Edexcel 9FM0: FM2 Further centres of mass (laminas and non-uniform rods by integration)

HOMEWORK Consolidation

1 A uniform lamina is bounded by $y = 2x^3$, the x-axis and the line $x = 2$. Find its centre of mass.

AQA 7367 ME3 · Edexcel FM2 (Further centres of mass: lamina by integration)

2 A uniform lamina is bounded by $y = 8 - 2x^2$ and the coordinate axes, with $x \geq 0$. Find its centre of mass.

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3 A uniform lamina is bounded by $y = 1/x$, the x-axis and the lines $x = 1$ and $x = e$. Find its centre of mass, giving exact answers.

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4 A uniform lamina occupies the region between the curves $y = x^2$ and $y = 2 - x^2$. Use integration to show that $\bar{y} = 1$, and explain why this was to be expected.

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5 A uniform lamina occupies the region between the line $y = 3x$ and the curve $y = x^2$. Find its centre of mass.

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6 A uniform lamina is bounded by the curve $y^2 = x^3$ and the line $x = 4$. Find its centre of mass.

AQA 7367 ME3 (symmetry; standard results) · Edexcel FM2 (Further centres of mass)

7 A rod AB of length 6 m has mass per unit length $k(x + 3)$ kg m^{-1} at a distance x m from A, and its mass is 36 kg.

(a) Find k.

(b) Find the distance of its centre of mass from A.

Edexcel FM2 (Further centres of mass: non-uniform rod) · not in AQA 7367

8 A uniform lamina is bounded by $y = k(4x - x^2)$ $(k > 0)$ and the x-axis. Its centre of mass is at $(2, 2)$. Find k.

AQA 7367 ME3 · Edexcel FM2 (unknown constants)

9 A uniform lamina is bounded by $y = 2x^2$, the x-axis and the line $x = 2$. A is the point $(2, 0)$.

(a) Find its centre of mass.

(b) The lamina hangs freely from A. Find the angle between OA and the vertical.

AQA 7367 ME3, ME2 (freely suspended) · Edexcel FM2 (lamina by integration, hanging)

10 A uniform lamina is bounded by $y = \sin x$ and the x-axis for $0 \leq x \leq \pi$. Find its centre of mass, giving exact answers.

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11 *AQA-style*. A uniform lamina is bounded by the curve $y = 1 + x^2$, the coordinate axes and the line $x = 3$.

(a) Show that $\bar{x} = 33/16$. [3 marks]

(b) Find $\bar{y}$. [2 marks]

(c) The lamina hangs freely from O. Find the angle between its straight edge along the y-axis and the vertical. [2 marks]

AQA 7367 ME3, ME2 (freely suspended) · Edexcel FM2 (lamina by integration, hanging) · AQA-style

12 *Edexcel-style*. A non-uniform rod AB has length $2a$. At a distance x from A its mass per unit length is $k(a + x)$, where k is a constant. The mass of the rod is M.

(a) Show that the centre of mass of the rod is $\frac{7a}{6}$ from A. [4 marks]

(b) A particle is attached at A, so that the centre of mass of the loaded rod is at the midpoint of AB. Find the mass of the particle in terms of M. [2 marks]

Edexcel FM2 (Further centres of mass: non-uniform rod) · not in AQA 7367 · Edexcel-style