Laminas by Integration

Specification: AQA 7367: ME3 (lamina by integration) · Edexcel 9FM0: FM2 Further centres of mass (laminas and non-uniform rods by integration)

INDEPENDENT

You do

A · Under a curve A = ∫y dx, then $\bar{x}$ = ∫xy dx ÷ A and $\bar{y}$ = ∫½y² dx ÷ A.

1 A uniform lamina is bounded by $y = 2x$, the x-axis and $x = 3$. Find its centre of mass by integration.

2 A uniform lamina is bounded by $y = x^2$, the x-axis and $x = 2$. Find its centre of mass.

3 A uniform lamina is bounded by $y = 9 - x^2$ and the coordinate axes, with $x \geq 0$. Find its centre of mass.

4 A uniform lamina is bounded by $y = 1/x^2$, the x-axis and the lines $x = 1$ and $x = 2$. Find its centre of mass, giving $\bar{x}$ exactly.

B · Roots, two curves, signs and rods Strip between two curves: height $y_1 - y_2$, centre $\frac{1}{2}(y_1 + y_2)$.

5 A uniform lamina is bounded by $y = 2\sqrt{x}$, the x-axis and $x = 9$. Find its centre of mass.

6 A uniform lamina occupies the region between the curve $y = x^2$, the line $y = 4$ and the y-axis ($0 \leq x \leq 2$). Find its centre of mass.

7 A uniform lamina occupies the region between $y = x$ and $y = x^3$ for $0 \leq x \leq 1$. Find its centre of mass.

8 A uniform lamina occupies the region between the x-axis and the curve $y = x^2 - 4$. Find its centre of mass.

9 A rod AB of length 4 m has mass per unit length $(2 + x)$ kg m^{-1} at a distance x m from A. Find its mass and the distance of its centre of mass from A.

C · Reasoning No calculator. Explain in words.

10 A uniform lamina is bounded by $y = 4x - x^2$ and the x-axis. Without integrating, write down $\bar{x}$. Explain your answer.

11 A uniform lamina fills the region under a curve $y = f(x)$ ($f(x) \geq 0$), and the greatest value of $f(x)$ is 6. Explain why $\bar{y} \leq 3$. When is $\bar{y} = 3$?

12 A uniform lamina R lies under a curve with area 3, $\int xy\,dx = 4$ and $\int \frac{1}{2}y^2\,dx = 2.4$. A second lamina is R stretched by scale factor 2 parallel to the y-axis (every y doubled). Find the centre of mass of each.

D · Show that Keep the density ρ in a proof. Show every integral.

13 A uniform triangular lamina of density ρ is bounded by the line $y = hx/b$, the x-axis and the line $x = b$. Show by integration that its centre of mass is at $(\frac{2}{3}b, \frac{1}{3}h)$.

14 A uniform lamina is bounded by $y = x^n$ ($n > 0$), the x-axis and the line $x = 1$. Show that $\bar{x} = \dfrac{n+1}{n+2}$. What happens to $\bar{x}$ as n gets large? Explain.

15 A uniform lamina is bounded by $y = e^x$, the coordinate axes and the line $x = 1$. Show that $\bar{x} = \dfrac{1}{e-1}$ and $\bar{y} = \dfrac{e+1}{4}$.

E · Exam-style question Show your method: marks are for working.

16 A uniform lamina occupies the region between the curve $y = 6x - x^2$ and the line $y = 2x$. The line meets the curve at O and at A.

(a) Show that the area of the lamina is $32/3$. [2 marks]

(b) Explain why $\bar{x} = 2$. [1 mark]

(c) Show that $\bar{y} = 28/5$. [3 marks]

(d) The lamina hangs freely from O. Find the angle between OA and the vertical. [2 marks]