Solids of Revolution by Integration

Specification: AQA 7367: ME4 (solids of revolution by integration) · Edexcel 9FM0: FM2 Further centres of mass (uniform and non-uniform solids of revolution)

HOMEWORK Consolidation

1 The region under $y = x^2 + 1$ from $x = 0$ to $x = 1$ is rotated about the x-axis. Find $\bar{x}$.

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2 The region under $y = 2\sqrt{x}$ between $x = 1$ and $x = 4$ is rotated about the x-axis. Find $\bar{x}$, the x-coordinate of its centre of mass.

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3 The region under $y = 1/\sqrt{x}$ from $x = 1$ to $x = 4$ is rotated about the x-axis. Find $\bar{x}$, the x-coordinate of its centre of mass, exactly.

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4 A uniform frustum is formed by rotating $y = 2 + \frac{1}{2}x$, $0 \le x \le 2$, about the x-axis. Find the distance of its centre of mass from its smaller face.

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5 The cone formed by rotating $y = x/2$, $0 \le x \le 4$ ($V = 16\pi/3$, $\bar{x} = 3$) has a uniform cylinder of radius 2 and length 2 fixed to its base. Find the distance of the centre of mass from the vertex O.

AQA 7367 ME4, ME2 (composite bodies) · Edexcel FM2 (composite solids)

6 The region under $y = \sqrt{x}$ from $x = 0$ to $x = 1$ is rotated about the x-axis, and a uniform hemisphere of radius 1 is fixed to the plane face of the solid, so it runs from $x = 0$ to $x = 2$. Find $\bar{x}$ of the whole solid.

AQA 7367 ME4, ME2 (composite bodies) · Edexcel FM2 (composite solids)

7 The region under $y = \sqrt{(2x)}$ from $x = 0$ to $x = 2$ is rotated about the x-axis.

(a) Find $\bar{x}$.

(b) The solid hangs freely from a point on the rim of its plane face. Find the angle between its axis and the vertical.

AQA 7367 ME4, ME2 (freely suspended) · Edexcel FM2 (solids: hanging)

8 The hemisphere formed by rotating $y = \sqrt{(1 - x^2)}$, $0 \le x \le 1$, about the x-axis has density $\rho = k(1 + x)$, where x is the distance from the plane face. Find the distance of its centre of mass from O, the centre of its plane face.

Edexcel FM2 (Further centres of mass: non-uniform solid) · not in AQA 7367

9 The solid of Explore Q1 ($y = \sqrt{x}$, $0 \le x \le 4$, $V = 8\pi$, $\bar{x} = 8/3$) has a uniform cylinder of radius 2 and length L fixed to its plane face. The centre of mass of the combined solid is on the joined face, at $x = 4$. Find L.

AQA 7367 ME4 · Edexcel FM2 (unknown lengths)

10 Show that a uniform solid hemisphere hanging freely from a point on the rim of its plane face has its axis at $\tan^{-1}(8/3) = 69.4°$ to the vertical, whatever its radius.

AQA 7367 ME4 (standard results by integration) · Edexcel FM2 (Further centres of mass)

11 *AQA-style*. The region bounded by the curve $y = 4/x$, the x-axis and the lines $x = 1$ and $x = 2$ is rotated through 2π about the x-axis to form a uniform solid.

(a) Show that the volume of the solid is 8π. [2 marks]

(b) Show that $\bar{x} = 2 \ln 2$. [3 marks]

(c) The solid hangs freely from a point on the rim of its smaller plane face. Find the angle between its axis and the vertical. [2 marks]

AQA 7367 ME4, ME2 (freely suspended) · Edexcel FM2 (solids: hanging) · AQA-style

12 *Edexcel-style*. A solid S is formed by rotating the region under $y = \sqrt{x}$, $0 \le x \le a$, through 2π about the x-axis. The density of S at the cross-section a distance x from O is $k(a + x)$, where k is a constant.

(a) Show that the centre of mass of S is $\frac{7a}{10}$ from O. [4 marks]

(b) A particle whose mass is equal to the mass of S is attached to the centre of the plane face of S. Find the distance of the centre of mass of the loaded solid from O. [2 marks]

Edexcel FM2 (Further centres of mass: non-uniform solid) · not in AQA 7367 · Edexcel-style