Solids of Revolution by Integration

Specification: AQA 7367: ME4 (solids of revolution by integration) · Edexcel 9FM0: FM2 Further centres of mass (uniform and non-uniform solids of revolution)

INDEPENDENT

You do

A · By integration V = ∫πy² dx, then x̄ = ∫πxy² dx ÷ V. Square y first.

1 The region under $y = 2x$ from $x = 0$ to $x = 3$ is rotated about the x-axis. Find $\bar{x}$ of the solid cone.

2 The region under $y = x^2$ from $x = 0$ to $x = 1$ is rotated about the x-axis. Find $\bar{x}$.

3 The region under $y = 1/x$ from $x = 1$ to $x = 3$ is rotated about the x-axis. Find $\bar{x}$ exactly.

4 The region under $y = \sqrt{(x+1)}$ from $x = 0$ to $x = 3$ is rotated about the x-axis. Find $\bar{x}$.

B · Frustums, composites, hanging and density Composite: each part's V and x̄, then ΣVx ÷ ΣV. Hanging from a rim: tan θ = r ÷ d.

5 The region under $y = 1 + \frac{1}{2}x$ from $x = 0$ to $x = 4$ is rotated about the x-axis to form a frustum. Find the distance of its centre of mass from the smaller face.

6 The solid of Explore Q1 ($V = 8\pi$, $\bar{x} = 8/3$) has a uniform cylinder of radius 2 and length 2 fixed to its plane face: it runs from $x = 0$ to $x = 6$.
(a) Find $\bar{x}$.
(b) The solid is freely suspended from a point on the rim of its end face at $x = 6$. Find the angle between its axis and the vertical.

7 The solid cone formed by rotating $y = x$, $0 \le x \le 3$, about the x-axis has density $\rho = kx$, where x is the distance from the vertex O. Find the distance of its centre of mass from O.

8 The solid of Explore Q1 ($V = 8\pi$, $\bar{x} = 8/3$) has a uniform hemisphere of radius 2 fixed to its plane face: it runs from $x = 0$ to $x = 6$. Find $\bar{x}$.

9 The region under $y = x^2$ from $x = 0$ to $x = a$ is rotated about the x-axis. The centre of mass of the solid is 5 from O. Find a.

C · Reasoning No calculator. Explain in words.

10 The region under $y = x^2$ from $x = -1$ to $x = 1$ is rotated about the x-axis. Without integrating, write down $\bar{x}$. Explain.

11 A pupil says: "Stretching the region by scale factor 3 parallel to the y-axis moves the centre of mass of the solid of revolution." True or false? Explain.

12 A solid cone and a triangular lamina have the same cross-section through the axis (height h). The cone's G is $h/4$ from its base; the triangle's is $h/3$. Explain why the cone's G is nearer the base.

D · Show that Keep ρ and π in a proof. Show every integral.

13 A uniform frustum of density ρ is formed by rotating $y = r(1 + x/h)$, $0 \le x \le h$, about the x-axis (radii r and $2r$). Show that its centre of mass is $\frac{11h}{28}$ from its larger face.

14 The region under $y = x^n$ ($n > 0$) from $x = 0$ to $x = 1$ is rotated about the x-axis. Show that $\bar{x} = \frac{2n+1}{2n+2}$. Check your result for $n = 1$ (a cone).

15 The region under $y = e^x$ from $x = 0$ to $x = 1$ is rotated about the x-axis. Show that $\bar{x} = \frac{e^2+1}{2(e^2-1)}$.

E · Exam-style question Show your method: marks are for working.

16 The region bounded by the curve $y^2 = 3x$ ($y \ge 0$), the x-axis and the line $x = 3$ is rotated through 2π about the x-axis to form a uniform solid paraboloid P with its vertex at O.
(a) Show that the volume of P is $27\pi/2$. [2 marks]
(b) Show that the centre of mass of P is 2 from O. [2 marks]
(c) A uniform cylinder of the same material, radius 3 and length 2, is fixed to the plane face of P, so the combined solid runs from $x = 0$ to $x = 5$. Find the distance of its centre of mass from O. [2 marks]
(d) The combined solid hangs freely from a point A on the rim of the cylinder's free end. Find the angle between its axis and the vertical. [2 marks]