Sliding or Toppling under an Applied Force

Specification: AQA 7367: ME5 (sliding and toppling) · Edexcel 9FM0: FM2 Further centres of mass (sliding and toppling)

INDEPENDENT

You do

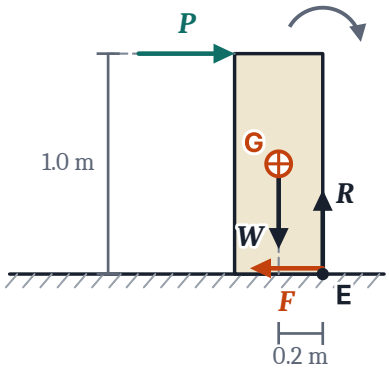
A · Slide or topple first? Slides at μR. Topples when P × h = W × d. The smaller force wins.

1 A uniform block, 0.4 m wide and 1.0 m tall, of weight 100 N, stands on rough horizontal ground (μ = 0.3). A horizontal force P acts at the top edge of its back face. Does it slide or topple first, and at what value of P?

Slides at $\mu R = 0.3 \times 100 = 30$ N — limiting friction

Topples: $P \times 1.0 = 100 \times 0.2$, $P = 20$ N — moments about E

20 < 30: it topples first, at 20 N — smaller force first



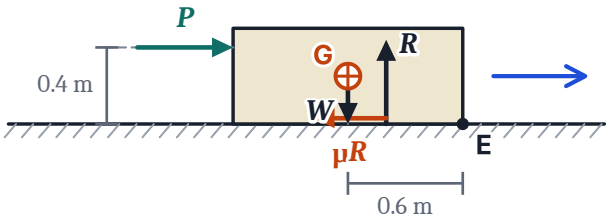
picture: topples

2 A uniform block, 1.2 m wide and 0.5 m tall, of weight 240 N, stands on rough horizontal ground (μ = 0.5). A horizontal force P acts on its back face 0.4 m above the ground. Does it slide or topple first, and at what value of P?

Slides at $0.5 \times 240 = 120$ N — limiting friction

Topples: $P \times 0.4 = 240 \times 0.6$, $P = 360$ N — moments about E

120 < 360: it slides first, at 120 N — smaller force first



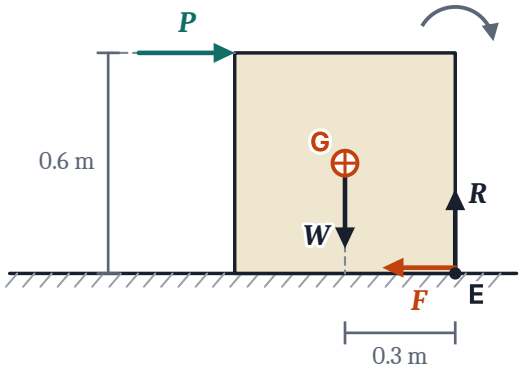
picture: slides

3 A uniform cube of side 0.6 m and mass 25 kg stands on rough horizontal ground (μ = 0.6). A horizontal force P acts at the middle of the top edge of one face, at right angles to that face. Does it slide or topple first, and at what value of P? Take $g = 9.8$ m s^{-2}.

$W = 25 \times 9.8 = 245$ N. Slides at $0.6 \times 245 = 147$ N — W = mg; limiting friction

Topples: $P \times 0.6 = 245 \times 0.3$, $P = 122.5$ N — moments about E

122.5 < 147: it topples first, at 122.5 N — smaller force first



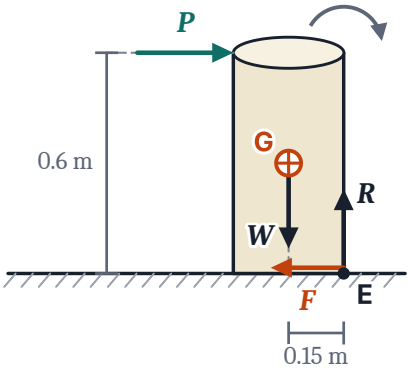
picture: topples

4 A uniform solid cylinder of weight 80 N, radius 0.15 m and height 0.6 m stands on its plane face on rough horizontal ground (μ = 0.3). A horizontal force P acts at a point on the rim of its top face, in the vertical plane through the axis. Does it slide or topple first, and at what value of P?

Slides at $0.3 \times 80 = 24$ N — limiting friction

Topples: $d = r = 0.15$, $P \times 0.6 = 80 \times 0.15$, $P = 20$ N — moments about E: d = r

20 < 24: it topples first, at 20 N — smaller force first



picture: topples

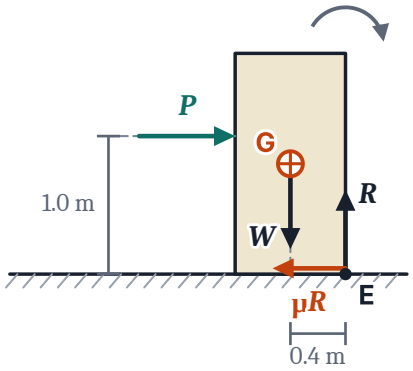
B · Ranges, angles and composite shapes Range: write the condition as an inequality, then check the force acts on the body. Angle: R = W − P sin α, and both parts of P can turn the body.

5 A uniform block of weight W, 0.8 m wide and 1.6 m tall, stands on rough horizontal ground ($\mu = 0.4$). A horizontal force pushes its back face at a height of h metres. Find the range of values of h for which it slides before it topples.

Slides at $0.4W$; topples at $W \times 0.4/h$ — P for each

Slides first if $0.4W < 0.4W/h$: $h < 1.0$. So $0 < h < 1.0$

slides first: smaller force



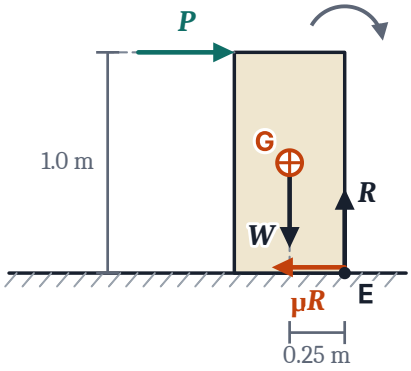
picture: at h = 1.0 m both together

6 A uniform block, 0.5 m wide and 1.0 m tall, stands on rough horizontal ground. A horizontal force acts at the top edge of its back face. Find the range of values of μ for which it slides before it topples.

Slides at μW; topples at $W \times 0.25/1.0 = 0.25W$ — P for each

Slides first if $\mu W < 0.25W$: $\mu < 0.25$

slides first: μ < d/h



picture: at μ = 0.25 both together

7 A uniform block, 0.6 m wide and 0.9 m tall, of weight 180 N, stands on rough horizontal ground ($\mu = 0.5$). A rope attached to the top edge of its front face pulls forwards at 40° above the horizontal. Does it slide or topple first, and at what tension?

Slides: $P\cos 40° = 0.5(180 - P\sin 40°)$,
$P(\cos 40° + 0.5\sin 40°) = 90$, $P = 82.8$ N

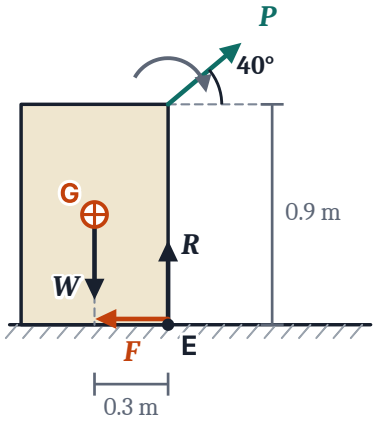
R = W − P sin α; F = μR

Topples: $P\cos 40° \times 0.9 = 180 \times 0.3$, $P = 78.3$ N

moments about E

78.3 < 82.8: it topples first, at 78.3 N

smaller force first



picture: topples

8 A uniform block, 1.0 m wide and 0.8 m tall, of weight 200 N, stands on rough horizontal ground ($\mu = 0.5$). A force P acts at the top edge of its back face, at 25° above the horizontal, towards the front edge. Does it slide or topple first, and at what value of P?

Slides: $P\cos 25° = 0.5(200 - P\sin 25°)$,
$P(\cos 25° + 0.5\sin 25°) = 100$, $P = 89.5$ N

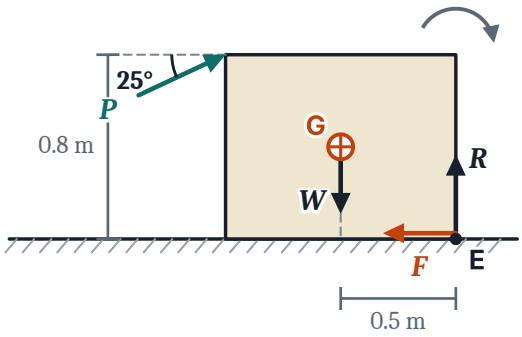
R = W − P sin α; F = μR

Topples: $P(0.8\cos 25° + 1.0\sin 25°) = 200 \times 0.5$, $P = 87.1$ N

moments about E: both parts

87.1 < 89.5: it topples first, at 87.1 N

smaller force first



picture: topples

9 A uniform lamina is a square of side 0.6 m with a rectangle 0.3 m wide and 0.6 m tall standing on the left half of its top edge. It has weight 90 N and stands in a vertical plane on the bottom of the square, on rough horizontal ground ($\mu = 0.4$).

(a) Find the distance of its centre of mass from the left-hand edge.

(b) A horizontal force P pushes to the right at the top left corner. Does it slide or topple first, and at what value of P?

(c) Instead, a horizontal force pushes to the left at the top of the right-hand edge of the square. Does it slide or topple first, and at what value?

(a)

	A	x	Ax
square	0.36	0.3	0.108
top	0.18	0.15	0.027
Σ	**0.54**		**0.135**

table: A, x, Ax

$\bar{x} = 0.135 \div 0.54 = 0.25$ m

ΣAx ÷ ΣA

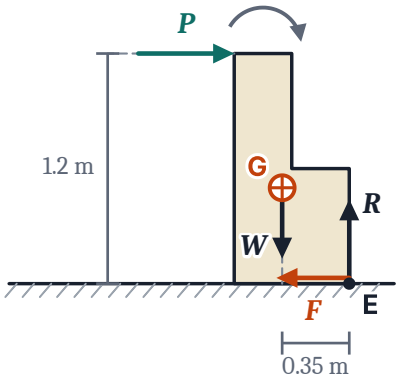
(b) Slides at $0.4 \times 90 = 36$ N. Front edge: right, $d = 0.6 - 0.25 = 0.35$: $P \times 1.2 = 90 \times 0.35$, $P = 26.25$ N. Topples first, at 26.25 N

front edge right: d = 0.35

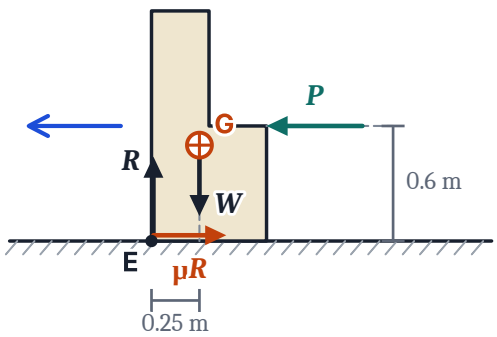
(c) Front edge: left, $d = 0.25$, height 0.6: $P \times 0.6 = 90 \times 0.25$, $P = 37.5$ N > 36. Slides first, at 36 N

front edge left: d = 0.25

(b)



(c)



picture: (b) topples, (c) slides

C · Reasoning No calculator. Explain in words.

10 A uniform solid cone and a uniform solid cylinder have the same weight, the same base radius r and the same height h. Each stands on its base on rough horizontal ground, and a horizontal force acts at its top (the vertex of the cone; a point on the rim of the cylinder's top face). A pupil says: "The cone is harder to topple, because its centre of mass is lower ($h/4$ against $h/2$)." Is the pupil right? Explain.

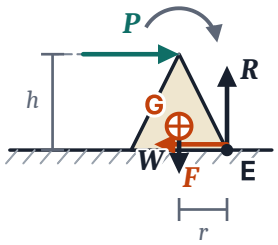
Moments about E: $P \times h = W \times r$ for both, because each G is on the axis, r from E.

moments about E

The height of G never appears: $P = Wr/h$ for both. The pupil is wrong (on level ground).

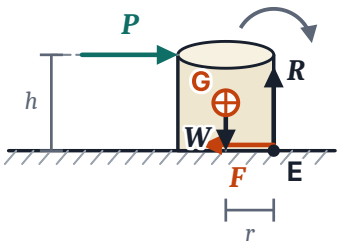
height of G does not matter

cone



picture: the same arms h and r

cylinder



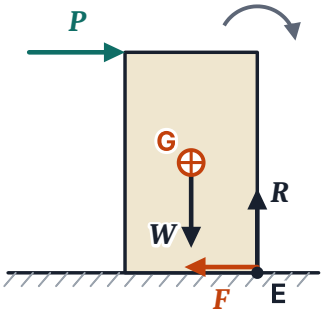
11 Explain why, when you take moments about the front edge E for a body on rough horizontal ground on the point of toppling, neither the normal reaction nor the friction appears in the equation.

On the point of toppling the body is about to lift off everywhere except at E, so the whole reaction acts at E. Friction acts along the ground at E.

R and F at E

Both forces pass through E, so their moments about E are zero.

zero moment



picture: R and F at E

12 A block stands on rough horizontal ground. A horizontal force acts on it at height h. The height h is doubled (the force still acts on the block). Without calculating, say what happens to
(a) the force needed to make it topple
(b) the force needed to make it slide.

(a) $P = Wd/h$: doubling h halves the toppling force.

P = Wd/h

(b) $P = \mu W$: the height plays no part in resolving, so it is unchanged.

P = μW

D · Show that Keep letters. Write each condition as an inequality and simplify.

13 A uniform block of weight W, $2a$ wide and $6a$ tall, stands on rough horizontal ground (coefficient of friction μ). A horizontal force pushes its back face at height h, where $0 < h \leq 6a$.

(a) Show that it slides before it topples if $h < a/\mu$.

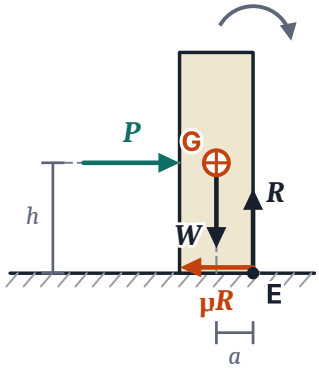
(b) Show that if $\mu < 1/6$ it slides first wherever the force acts.

(a) Slides at μW; topples at Wa/h. Slides first if $\mu W < Wa/h$, i.e. $h < a/\mu$

compare: μW < Wd/h

(b) If $\mu < 1/6$ then $a/\mu > 6a \geq h$, so $h < a/\mu$ for every height: it slides first.

a/μ > 6a



picture: moments about E

14 A uniform block of weight W, $2a$ wide and $3a$ tall, stands on rough horizontal ground (coefficient of friction μ). A rope attached to the top edge of its front face pulls forwards at angle θ above the horizontal, where $\tan\theta < 3$. Show that it topples before it slides if $\mu > 1/(3 - \tan\theta)$.

Slides at $P = \mu W/(\cos\theta + \mu\sin\theta)$ (resolve: $R = W - P\sin\theta$).

resolve; F = μR

Moments about E (the vertical component of P passes through E): $P\cos\theta \times 3a = W \times a$, $P = W/(3\cos\theta)$

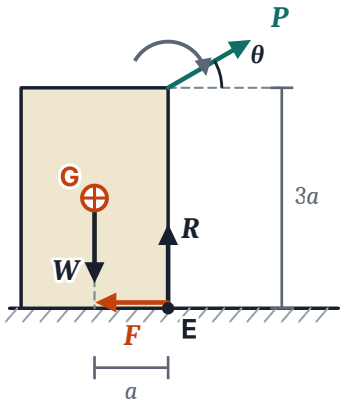
moments about E

Topples first if $W/(3\cos\theta) < \mu W/(\cos\theta + \mu\sin\theta)$: $\cos\theta + \mu\sin\theta < 3\mu\cos\theta$, $\cos\theta < \mu(3\cos\theta - \sin\theta)$

smaller force first

Divide by $\cos\theta > 0$: $1 < \mu(3 - \tan\theta)$. As $3 - \tan\theta > 0$: $\mu > 1/(3 - \tan\theta)$

÷ cos θ



picture: moments about E

15 A uniform lamina is bounded by the curve $y = 4 - x^2$, the x-axis and the y-axis ($x \geq 0$). It stands in a vertical plane with its straight edge along the x-axis on rough horizontal ground (coefficient of friction μ). A horizontal force acts at its highest point $(0, 4)$, pushing in the direction of x increasing.

(a) Show that $\bar{x} = 3/4$.

(b) Show that it topples before it slides if $\mu > 5/16$.

(a) $A = \int_0^2 (4 - x^2)\,dx = 16/3$, $\int_0^2 x(4 - x^2)\,dx = \left[2x^2 - x^4/4\right]_0^2 = 4$

area; $\int xy\,dx$

$\bar{x} = 4 \div (16/3) = 3/4$

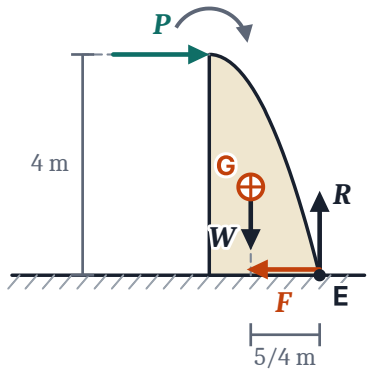
÷ A

(b) Front edge $(2, 0)$: $d = 2 - 3/4 = 5/4$. Topples when $P \times 4 = W \times 5/4$, $P = 5W/16$

moments about the front edge

Slides at μW. Topples first if $5W/16 < \mu W$: $\mu > 5/16$

smaller force first



picture: moments about (2, 0)

E · Exam-style question Show your method: marks are for working.

16 A uniform lamina OABC has vertices O$(0, 0)$, A$(1.5, 0)$, B$(1.5, 0.6)$, C$(0, 1.2)$ (metres): a rectangle with a triangle on top.

(a) Show that its centre of mass is $\frac{2}{3}$ m from OC. [3]

It weighs 60 N and stands in a vertical plane with OA on rough horizontal ground. A horizontal force P acts at C, in the direction OA.

(b) Find P on the point of toppling about A. [2]

(c) Given that $\mu = 0.4$, show that the lamina slides before it topples. [2]

(d) Find the range of values of μ for which it would topple first. [1]

(a) Rectangle: $A = 0.9$ at $x = 0.75$; triangle: $A = 0.45$ at $x = (0 + 1.5 + 0)/3 = 0.5$ **B1** — areas; triangle: mean of the vertices

$\Sigma A = 1.35$, $\Sigma Ax = 0.675 + 0.225 = 0.9$ **M1** — add the columns

$\bar{x} = 0.9 \div 1.35 = 2/3$ m **A1** — ΣAx ÷ ΣA

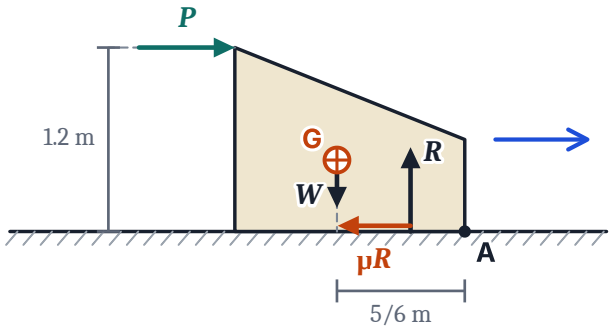
(b) $d = 1.5 - 2/3 = 5/6$: $P \times 1.2 = 60 \times 5/6$ **M1** — moments about A

$P = 125/3 = 41.7$ N **A1** — solve

(c) $\mu R = 0.4 \times 60 = 24$ N **M1** — limiting friction

$24 < 41.7$: it slides first. **A1** — compare

(d) $60\mu > 125/3$: $\mu > 25/36 = 0.694$ **B1** — μW > P to topple



picture: slides first