Sliding or Toppling under an Applied Force

Specification: AQA 7367: ME5 (sliding and toppling) · Edexcel 9FM0: FM2 Further centres of mass (sliding and toppling)

INDEPENDENT

You do

A · Slide or topple first? Slides at μR. Topples when P × h = W × d. The smaller force wins.

1 A uniform block, 0.4 m wide and 1.0 m tall, of weight 100 N, stands on rough horizontal ground ($\mu = 0.3$). A horizontal force P acts at the top edge of its back face. Does it slide or topple first, and at what value of P?

2 A uniform block, 1.2 m wide and 0.5 m tall, of weight 240 N, stands on rough horizontal ground ($\mu = 0.5$). A horizontal force P acts on its back face 0.4 m above the ground. Does it slide or topple first, and at what value of P?

3 A uniform cube of side 0.6 m and mass 25 kg stands on rough horizontal ground ($\mu = 0.6$). A horizontal force P acts at the middle of the top edge of one face, at right angles to that face. Does it slide or topple first, and at what value of P? Take $g = 9.8$ m s^{-2}.

4 A uniform solid cylinder of weight 80 N, radius 0.15 m and height 0.6 m stands on its plane face on rough horizontal ground ($\mu = 0.3$). A horizontal force P acts at a point on the rim of its top face, in the vertical plane through the axis. Does it slide or topple first, and at what value of P?

B · Ranges, angles and composite shapes Range: write the condition as an inequality, then check the force acts on the body. Angle: R = W − P sin α, and both parts of P can turn the body.

5 A uniform block of weight W, 0.8 m wide and 1.6 m tall, stands on rough horizontal ground ($\mu = 0.4$). A horizontal force pushes its back face at a height of h metres. Find the range of values of h for which it slides before it topples.

6 A uniform block, 0.5 m wide and 1.0 m tall, stands on rough horizontal ground. A horizontal force acts at the top edge of its back face. Find the range of values of μ for which it slides before it topples.

7 A uniform block, 0.6 m wide and 0.9 m tall, of weight 180 N, stands on rough horizontal ground ($\mu = 0.5$). A rope attached to the top edge of its front face pulls forwards at $40°$ above the horizontal. Does it slide or topple first, and at what tension?

8 A uniform block, 1.0 m wide and 0.8 m tall, of weight 200 N, stands on rough horizontal ground ($\mu = 0.5$). A force P acts at the top edge of its back face, at $25°$ above the horizontal, towards the front edge. Does it slide or topple first, and at what value of P?

9 A uniform lamina is a square of side 0.6 m with a rectangle 0.3 m wide and 0.6 m tall standing on the left half of its top edge. It has weight 90 N and stands in a vertical plane on the bottom of the square, on rough horizontal ground ($\mu = 0.4$).
(a) Find the distance of its centre of mass from the left-hand edge.
(b) A horizontal force P pushes to the right at the top left corner. Does it slide or topple first, and at what value of P?
(c) Instead, a horizontal force pushes to the left at the top of the right-hand edge of the square. Does it slide or topple first, and at what value?

C · Reasoning No calculator. Explain in words.

10 A uniform solid cone and a uniform solid cylinder have the same weight, the same base radius r and the same height h. Each stands on its base on rough horizontal ground, and a horizontal force acts at its top (the vertex of the cone; a point on the rim of the cylinder's top face). A pupil says: "The cone is harder to topple, because its centre of mass is lower ($h/4$ against $h/2$)." Is the pupil right? Explain.

11 Explain why, when you take moments about the front edge E for a body on rough horizontal ground on the point of toppling, neither the normal reaction nor the friction appears in the equation.

12 A block stands on rough horizontal ground. A horizontal force acts on it at height h. The height h is doubled (the force still acts on the block). Without calculating, say what happens to
(a) the force needed to make it topple
(b) the force needed to make it slide.

D · Show that Keep letters. Write each condition as an inequality and simplify.

13 A uniform block of weight W, $2a$ wide and $6a$ tall, stands on rough horizontal ground (coefficient of friction μ). A horizontal force pushes its back face at height h, where $0 < h \le 6a$.
(a) Show that it slides before it topples if $h < a/\mu$.
(b) Show that if $\mu < 1/6$ it slides first wherever the force acts.

14 A uniform block of weight W, $2a$ wide and $3a$ tall, stands on rough horizontal ground (coefficient of friction μ). A rope attached to the top edge of its front face pulls forwards at angle θ above the horizontal, where $\tan\theta < 3$. Show that it topples before it slides if $\mu > 1/(3 - \tan\theta)$.

15 A uniform lamina is bounded by the curve $y = 4 - x^2$, the x-axis and the y-axis ($x \ge 0$). It stands in a vertical plane with its straight edge along the x-axis on rough horizontal ground (coefficient of friction μ). A horizontal force acts at its highest point $(0, 4)$, pushing in the direction of x increasing.
(a) Show that $\bar{x} = 3/4$.
(b) Show that it topples before it slides if $\mu > 5/16$.

E · Exam-style question Show your method: marks are for working.

16 A uniform lamina OABC has vertices O$(0, 0)$, A$(1.5, 0)$, B$(1.5, 0.6)$, C$(0, 1.2)$ (metres): a rectangle with a triangle on top.

(a) Show that its centre of mass is $\frac{2}{3}$ m from OC. [3]

It weighs 60 N and stands in a vertical plane with OA on rough horizontal ground. A horizontal force P acts at C, in the direction OA.
(b) Find P on the point of toppling about A. [2]
(c) Given that $\mu = 0.4$, show that the lamina slides before it topples. [2]
(d) Find the range of values of μ for which it would topple first. [1]