Sliding or Toppling on an Inclined Plane

Specification: AQA 7367: ME5 (sliding and toppling) · Edexcel 9FM0: FM2 Further centres of mass (toppling of a lamina on an inclined plane; solids A level only)

INDEPENDENT You do

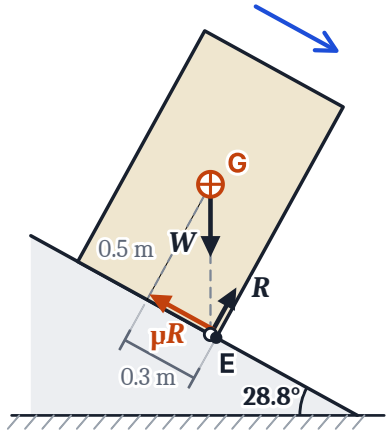
A · Slide or topple first? Slides: tan α = μ. Topples: tan α = d/h. The smaller angle wins.

1 A uniform block, 0.6 m wide (along the slope) and 1.0 m tall, stands on a rough plane (μ = 0.55) whose angle is slowly increased. Does it slide or topple first, and at what angle?

Slides: tan $\alpha = 0.55$, $\alpha = 28.8°$ — slides: tan α = μ

Topples: tan $\alpha = 0.3/0.5 = 0.6$, $\alpha = 31.0°$ — topples: tan α = d/h

28.8° < 31.0°: it slides first, at 28.8° — smaller angle first



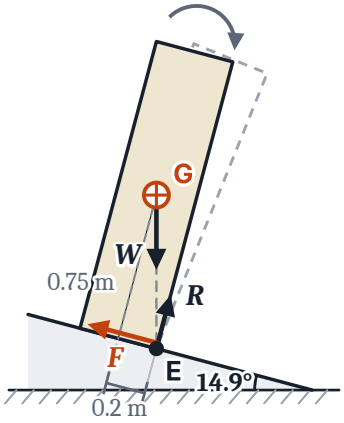
picture: slides

2 A uniform block, 0.4 m wide (along the slope) and 1.5 m tall, stands on a rough plane (μ = 0.45) whose angle is slowly increased. Does it slide or topple first, and at what angle?

Slides: tan $\alpha = 0.45$, $\alpha = 24.2°$ — slides: tan α = μ

Topples: tan $\alpha = 0.2/0.75 = 4/15$, $\alpha = 14.9°$ — topples: tan α = d/h

14.9° < 24.2°: it topples first, at 14.9° — smaller angle first



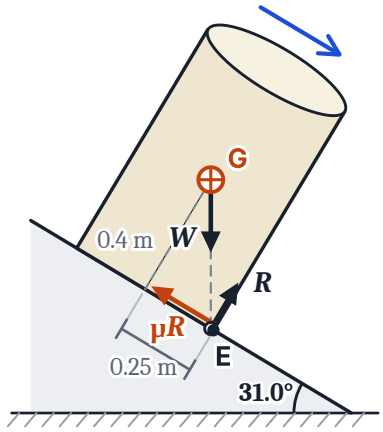
picture: topples

3 A uniform solid cylinder of radius 0.25 m and height 0.8 m stands on a plane face on a rough plane (μ = 0.6) whose angle is slowly increased. Does it slide or topple first, and at what angle?

Slides: tan $\alpha = 0.6$, $\alpha = 31.0°$ — slides: tan α = μ

Topples: $d = 0.25$, $h = 0.4$: tan $\alpha = 0.625$, $\alpha = 32.0°$ — topples: d = r, h = half the height

31.0° < 32.0°: it slides first, at 31.0° — smaller angle first



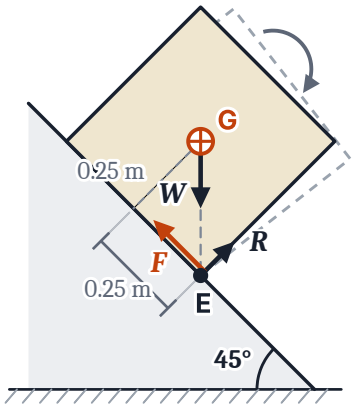
picture: slides

4 A uniform cube of side 0.5 m stands on a rough plane (μ = 1.2) whose angle is slowly increased. Does it slide or topple first, and at what angle?

Slides: tan $\alpha = 1.2$, $\alpha = 50.2°$ — slides: tan α = μ

Topples: tan $\alpha = 0.25/0.25 = 1$, $\alpha = 45°$ — topples: tan α = d/h

45° < 50.2°: it topples first, at 45° — smaller angle first



picture: topples

B · Ranges, composite bodies and solids

A range: write the condition as an inequality. Find G first (Units 1–2): h is its height above the plane, d its distance from the lowest edge along the slope.

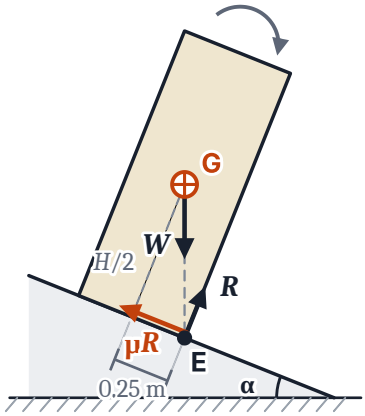
5 A uniform block, 0.5 m wide (along the slope) and H metres tall, stands on a rough plane ($\mu = 0.4$) whose angle is slowly increased. Find the range of values of H for which it slides before it topples.

Slides: $\tan\alpha = 0.4$. Topples: $\tan\alpha = 0.25 \div (H/2) = 0.5/H$

each condition

Slides first if $0.4 < 0.5/H$: $H < 1.25$. So $0 < H < 1.25$

slides first: $\mu < d/h$



picture: at H = 1.25 m both together

6 A rough plane is fixed at 25° to the horizontal. A uniform block, 0.4 m wide (along the slope) and H metres tall, is placed on it ($\mu = 0.8$).

(a) Show that the block does not slide.

(b) Find the greatest value of H for which it does not topple.

(a) $\tan 25° = 0.466 < 0.8$: friction can hold it.

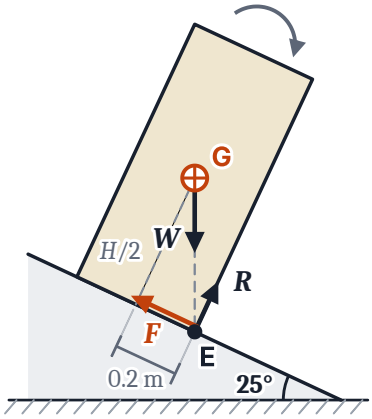
$\tan\alpha < \mu$

(b) It does not topple if $\tan 25° \le d/h = 0.2 \div (H/2) = 0.4/H$: $H \le 0.4 \div \tan 25°$

$\tan\alpha \le d/h$

$H \le 0.858$. Greatest $H = 0.858$ m

solve



picture: on the point of toppling at 25°

7 A uniform lamina is a rectangle 0.6 m wide and 0.3 m tall with an isosceles triangle of height 0.6 m on its top edge (the triangle's base is the whole top edge). It stands on the bottom edge of the rectangle in a vertical plane containing a line of greatest slope of a rough plane ($\mu = 0.8$) whose angle is slowly increased.

(a) Show that its centre of mass is 0.325 m above its base.

(b) Does it slide or topple first, and at what angle?

(a)

	A	y	Ay
rectangle	0.18	0.15	0.027
triangle	0.18	0.5	0.09
Σ	**0.36**		**0.117**

table: A, y, Ay

Triangle: $y = 0.3 + 0.6/3 = 0.5$

$\bar{y} = 0.117 \div 0.36 = 0.325$ m

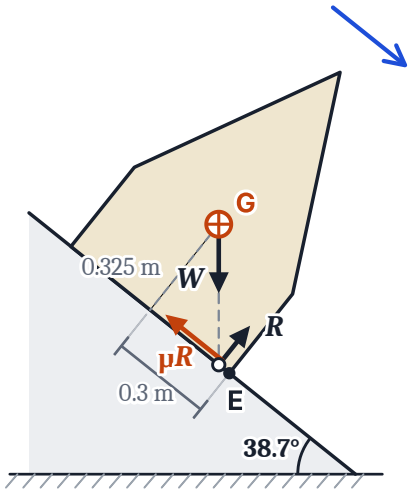
ΣAy ÷ ΣA

(b) By symmetry $d = 0.3$: topples when $\tan\alpha = 0.3/0.325 = 12/13$, $\alpha = 42.7°$. Slides when $\tan\alpha = 0.8$, $\alpha = 38.7°$

each angle

38.7° < 42.7°: it slides first, at 38.7°

smaller angle first



picture: slides

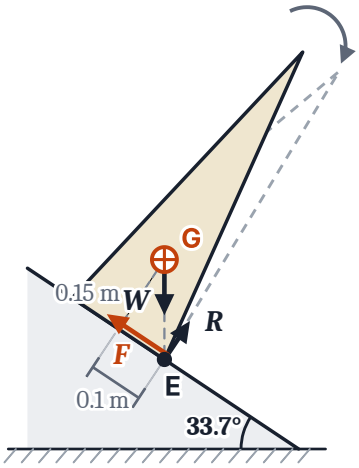
8 A uniform solid cone of base radius 0.1 m and height 0.6 m stands on its base on a rough plane ($\mu = 0.7$) whose angle is slowly increased. Does it slide or topple first, and at what angle?

G is $h/4 = 0.15$ m above the base: topples when $\tan\alpha = 0.1/0.15 = 2/3$, $\alpha = 33.7°$

cone: h/4; tan α = d/h

Slides when $\tan\alpha = 0.7$, $\alpha = 35.0°$. 33.7° < 35.0°: it topples first, at 33.7°

smaller angle first



picture: topples

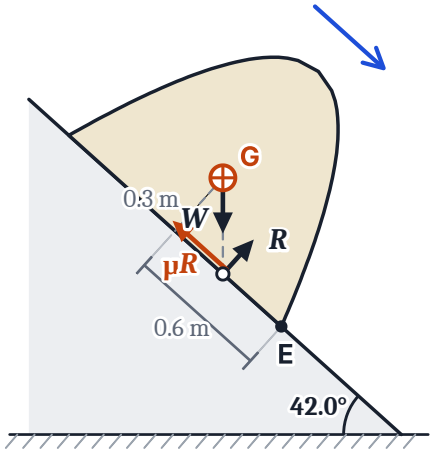
9 A uniform solid paraboloid (Lesson 2) has height 0.9 m and a plane face of radius 0.6 m; its centre of mass is on its axis, two-thirds of the height from the vertex. It stands on its plane face on a rough plane whose angle is slowly increased.
(a) Find the angle at which it would topple.
(b) Given $\mu = 0.9$, which happens first, and at what angle?

(a) G is $0.9 \div 3 = 0.3$ m above the plane face: $\tan\alpha = 0.6/0.3 = 2$, $\alpha = 63.4°$

h = a third of the height

(b) Slides when $\tan\alpha = 0.9$, $\alpha = 42.0°$. $42.0° < 63.4°$: it slides first, at $42.0°$

smaller angle first



picture: slides

C · Reasoning No calculator. Explain in words.

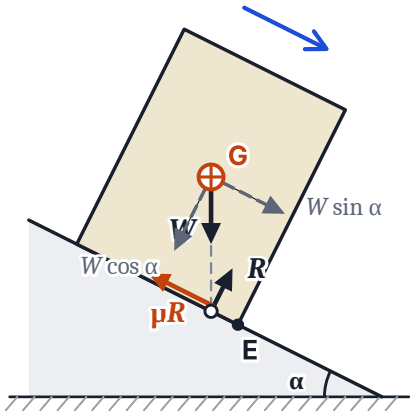
10 Two blocks have the same size and shape, but one is made of a denser material and weighs twice as much. They stand side by side on the same rough plane (same μ), whose angle is slowly increased. A pupil says: "The heavier block will start to slide at a smaller angle." Is the pupil right? Explain.

Sliding: $W \sin \alpha > \mu W \cos \alpha$, i.e. $\tan \alpha > \mu$: W cancels.

W cancels

Both start to slide at the same angle (and would topple at the same angle: d/h is the same). The pupil is wrong.

same angle



picture: R = W cos α, so W cancels

11 A short block and a tall block have the same width and stand on the same rough plane, whose angle is slowly increased. On level ground (Lesson 3) only the horizontal distance of G from the front edge mattered for toppling. Explain why the height of G matters on a slope, and say which block topples at the smaller angle.

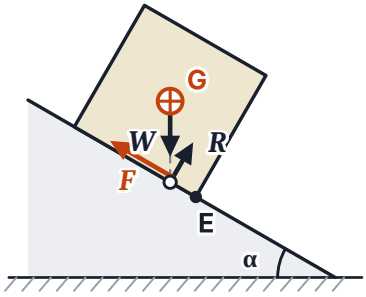
On a slope the weight has a part $W \sin \alpha$ down the slope, acting at G: its moment about E is $W \sin \alpha \times h$, which grows with the height of G.

W sin α × h

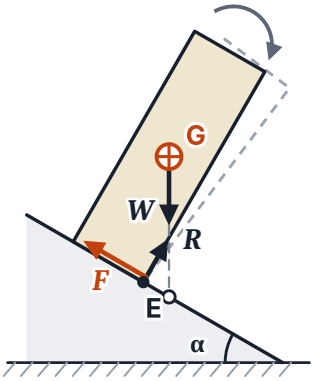
$\tan \alpha = d/h$: the larger h, the smaller the angle. The tall block topples at the smaller angle.

larger h, smaller angle

short



tall



picture: at 30° only the tall block topples

12 A uniform solid cylinder stands on a rough plane whose angle is slowly increased. It is replaced by a cylinder of the same radius and material but half the height. Without calculating, say what happens to
(a) the angle at which it would slide
(b) the angle at which it would topple.

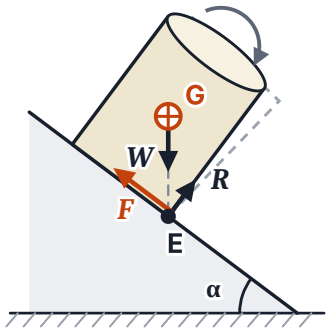
(a) $\tan\alpha = \mu$: the size plays no part. Unchanged.

tan α = μ

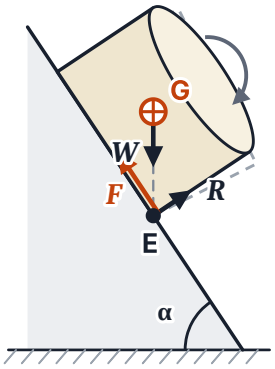
(b) $\tan\alpha = d/h = r \div (H/2)$: halving H doubles $\tan\alpha$, so the toppling angle is larger.

tan α = r ÷ (H/2)

height 0.8



height 0.4



picture: the shorter one leans further

D · Show that Keep letters. Write each condition as an inequality and simplify.

13 A uniform block of width $2a$ (along the slope) and height $2b$ stands on a rough plane (coefficient of friction μ) whose angle is slowly increased.

(a) Show that it topples before it slides if $b > a/\mu$.

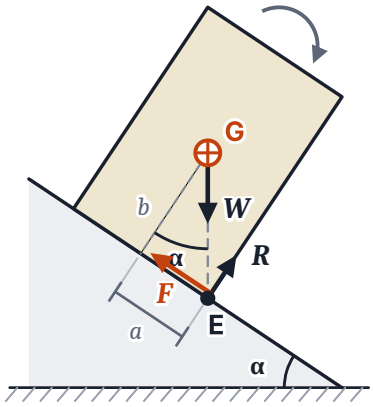
(b) A block is 0.5 m wide and $\mu = 0.4$. Find the range of heights for which it topples first.

(a) Slides when $\tan\alpha = \mu$; topples when $\tan\alpha = a/b$.

each condition

Topples first if $a/b < \mu$: $b > a/\mu$.

smaller angle first



picture: tan α = a/b on the point of toppling

(b) $a = 0.25$: $b > 0.25/0.4 = 0.625$, so the height $2b > 1.25$ m

substitute

14 A uniform solid cone of base radius r and height H stands on its base on a rough plane (coefficient of friction μ).

(a) Show that, as the angle of the plane is slowly increased, it topples before it slides if $H > 4r/\mu$.

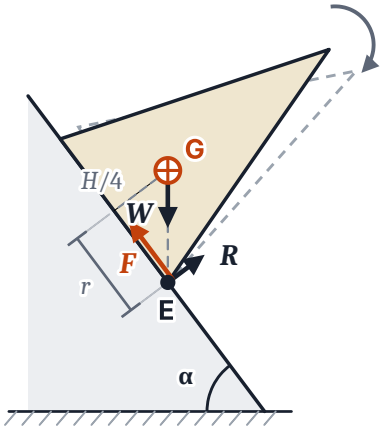
(b) A cone has base radius 5 cm and $\mu = 0.8$. Find the range of heights for which it topples first.

(a) G is $H/4$ above the base: topples when $\tan\alpha = r \div (H/4) = 4r/H$; slides when $\tan\alpha = \mu$

cone: H/4

Topples first if $4r/H < \mu$: $H > 4r/\mu$.

smaller angle first



picture: tan α = 4r/H on the point of toppling

(b) $H > 4 \times 5 \div 0.8 = 25$ cm

substitute

15 A box of weight W rests on a rough plane inclined at α (coefficient of friction μ). A horizontal force P, in the vertical plane through G containing a line of greatest slope, pushes the box towards the plane. The box does not topple.

(a) Show that it is about to slide up the plane when $P = W(\sin\alpha + \mu\cos\alpha)/(\cos\alpha - \mu\sin\alpha)$.

(b) Find P when $W = 100$ N, $\alpha = 20°$ and $\mu = 0.3$.

(a) $\perp$: $R = W\cos\alpha + P\sin\alpha$ (the horizontal force presses it onto the plane).

resolve $\perp$ the plane

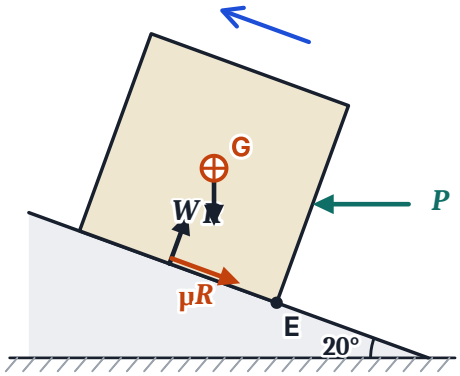
About to slide up, so friction acts down the slope. $\parallel$: $P\cos\alpha = W\sin\alpha + \mu(W\cos\alpha + P\sin\alpha)$

resolve $\parallel$; $F = \mu R$ down

$P(\cos\alpha - \mu\sin\alpha) = W(\sin\alpha + \mu\cos\alpha)$, so collect P

$$P = \frac{W(\sin\alpha + \mu\cos\alpha)}{\cos\alpha - \mu\sin\alpha}$$

rearrange



picture: about to slide up: friction down the slope

(b) $P = 100 \times 0.6239 \div 0.8371 = 74.5$ N

substitute

E · Exam-style question Show your method: marks are for working.

16 A uniform lamina OABC has vertices O$(0, 0)$, A$(0.9, 0)$, B$(0.9, 0.3)$ and C$(0, 0.9)$ (metres): a rectangle with a right-angled triangle on top.

(a) Show that its centre of mass is 0.375 m from OC and 0.325 m from OA. [4]

The lamina stands in a vertical plane with OA along a line of greatest slope of a rough plane. The angle of the plane is slowly increased.

(b) A is the lower end of OA. Assuming it does not slide first, find the angle at which the lamina would topple. [2]

(c) The lamina is turned round so that O is the lower end, and $\mu = 1.2$. Determine whether it slides or topples first. [2]

(a) Rectangle: $A = 0.27$ at $(0.45, 0.15)$; triangle $(0, 0.3)$, $(0.9, 0.3)$, $(0, 0.9)$: $A = 0.27$ at $(0.3, 0.5)$ **B1**

areas; triangle: mean of the vertices

$\Sigma A = 0.54$, $\Sigma Ax = 0.1215 + 0.081 = 0.2025$, $\Sigma Ay = 0.0405 + 0.135 = 0.1755$ **M1**

add the columns

$\bar{x} = 0.2025 \div 0.54 = 0.375$ m, $\bar{y} = 0.1755 \div 0.54 = 0.325$ m **A1 A1**

÷ ΣA

(b) $d = 0.9 - 0.375 = 0.525$, $h = 0.325$: $\tan\alpha = 0.525 \div 0.325 = 21/13$ **M1**

tan α = d/h

$\alpha = 58.2°$ **A1**

tan⁻¹

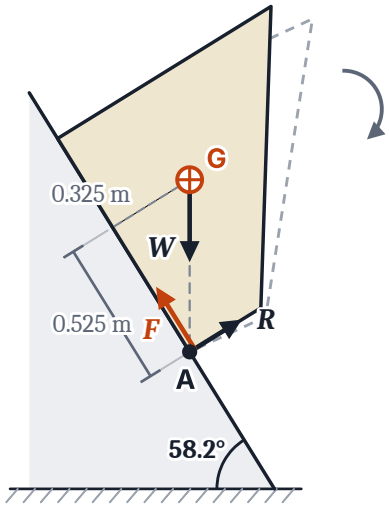
(c) Topples when $\tan\alpha = 0.375 \div 0.325 = 15/13$: $\alpha = 49.1°$; slides when $\tan\alpha = 1.2$: $\alpha = 50.2°$ **M1**

both angles

$49.1° < 50.2°$: it topples first. **A1**

compare

(b)



picture: (b) topples about A, (c) topples about O first

(c)

