# Sliding or Toppling on an Inclined Plane

Specification: AQA 7367: ME5 (sliding and toppling) · Edexcel 9FM0: FM2 Further centres of mass (toppling of a lamina on an inclined plane; solids A level only)

## HOMEWORK Consolidation

**1** A uniform block, $0.7$ m wide (along the slope) and $1.4$ m tall, stands on a rough plane ($\mu = 0.45$) whose angle is slowly increased. Does it slide or topple first, and at what angle?

AQA 7367 ME5 (sliding or toppling on an inclined plane) · Edexcel FM2 (toppling on an inclined plane)

**2** A uniform solid cylinder of radius $0.3$ m and height $1.2$ m stands on a plane face on a rough plane ($\mu = 0.6$) whose angle is slowly increased. Does it slide or topple first, and at what angle?

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**3** A uniform block, $0.8$ m wide (along the slope) and $H$ metres tall, stands on a rough plane ($\mu = 0.5$) whose angle is slowly increased. Find the range of values of $H$ for which it topples before it slides.

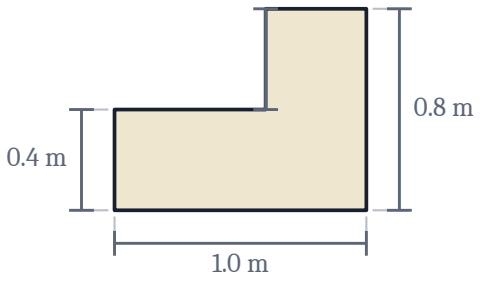
AQA 7367 ME5 (range of values) · Edexcel FM2 (toppling on an inclined plane)

**4** A rough plane is fixed at $35°$ to the horizontal. A uniform solid cone of height $0.4$ m stands on its base on the plane and does not slide. Find the least base radius for which it does not topple.

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**5** A uniform lamina is a rectangle $1.0$ m wide and $0.4$ m tall with a square of side $0.4$ m on top of its right-hand end. It stands on its long edge in a vertical plane containing a line of greatest slope of a rough plane ($\mu = 1.5$) whose angle is slowly increased, with the square on the downhill side.
(a) Find the distance of its centre of mass from the left-hand edge and from the base.
(b) Does it slide or topple first, and at what angle?

AQA 7367 ME5, ME2 (composite lamina) · Edexcel FM2 (lamina on an inclined plane)




**6** A uniform lamina is bounded by $y = \sqrt{x}$, the $x$-axis and the line $x = 4$ (lengths in metres). Its centre of mass has $\bar{x} = 12/5$ (Lesson 1).
(a) Show that $\bar{y} = 3/4$.
(b) It stands with its edge along the $x$-axis on a line of greatest slope of a rough plane, with $(4, 0)$ at the lower end. The angle of the plane is slowly increased. Find the angle at which it would topple.
(c) Find the range of values of $\mu$ for which it slides first.

AQA 7367 ME5, ME3 (lamina by integration) · Edexcel FM2 (Further centres of mass)

**7** A uniform solid hemisphere of radius $r$ rests with its plane face on a rough plane.
(a) Show that it would topple when $\tan\alpha = 8/3$, whatever its radius.
(b) Given $\mu = 0.75$, which happens first as the angle is slowly increased, and at what angle?

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**8** A uniform solid is a cylinder of radius $0.2$ m and height $0.6$ m with a hemisphere of radius $0.2$ m fixed to its top face. It stands on its bottom face on a rough plane ($\mu = 0.5$).
(a) Show that its centre of mass is $\frac{81}{220}$ m above the base.
(b) Which happens first as the angle is slowly increased, and at what angle?

AQA 7367 ME5, ME2 (uniform solids) · Edexcel FM2 (solids: A level only) · composite

**9** A uniform block, $0.4$ m wide (along the slope) and $0.6$ m tall, of weight $150$ N, rests on a rough plane inclined at $30°$ ($\mu = 0.25$). A force $P$ up the slope, parallel to the plane, acts at the middle of its downhill face ($0.3$ m above the plane), in the vertical plane through G. U is the upper edge of its base.

(a) Show that without $P$ it would slide but would not topple.

(b) Find the least $P$ that stops it sliding down.

(c) Find the greatest $P$ for equilibrium.

AQA 7367 ME5 (a force holding a body on a slope) · Edexcel FM2 (sliding and toppling)

**10** A uniform cube of side $0.5$ m and weight $80$ N rests on a rough plane inclined at $25°$ ($\mu = 0.2$).

(a) Show that on its own it would slide but would not topple.

(b) A horizontal force $P$, in the vertical plane through G containing a line of greatest slope, pushes it towards the plane. Find the least $P$ that stops it sliding down.

AQA 7367 ME5 (a force holding a body on a slope) · Edexcel FM2 (sliding and toppling) · horizontal force

**11** *AQA-style*. A uniform solid cylinder of radius $r$ and height $H$ stands on a plane face on a rough plane inclined at $\alpha$ to the horizontal; the coefficient of friction is $\mu$. The angle $\alpha$ is slowly increased.

(a) Show that the cylinder slides before it topples if $H < 2r/\mu$. [3]

(b) For a cylinder with $r = 0.15$ m, $H = 0.5$ m and $\mu = 0.5$, find the angle at which it first moves, and say how it moves. [2]

(c) The plane is now fixed at $30°$. Find, in terms of its weight $W$, the least force acting up the plane, parallel to it, that keeps this cylinder at rest. [2]

AQA 7367 ME5, ME2 (uniform solids) · Edexcel FM2 (solids: A level only) · AQA-style

**12** *Edexcel-style*. A uniform lamina is a square ABCD of side $2a$ with an isosceles triangle DCE outside the square on DC, E a distance $3a$ above DC. It stands in a vertical plane with AB along a line of greatest slope of a rough plane, B below A (coefficient of friction $\mu$).

(a) Show that its centre of mass is $\dfrac{13a}{7}$ from AB. [2]

(b) The angle $\alpha$ of the plane is slowly increased. Show that, if it does not slide first, the lamina is about to topple when $\tan\alpha = 7/13$. [2]

(c) Given that it slides first, find the range of values of $\mu$. [2]

AQA 7367 ME5, ME2 (composite lamina) · Edexcel FM2 (lamina on an inclined plane) · Edexcel-style