Sliding or Toppling on an Inclined Plane

Specification: AQA 7367: ME5 (sliding and toppling) · Edexcel 9FM0: FM2 Further centres of mass (toppling of a lamina on an inclined plane; solids A level only)

INDEPENDENT

You do

A · Slide or topple first? Slides: tan α = μ. Topples: tan α = d/h. The smaller angle wins.

1 A uniform block, 0.6 m wide (along the slope) and 1.0 m tall, stands on a rough plane ($\mu = 0.55$) whose angle is slowly increased. Does it slide or topple first, and at what angle?

2 A uniform block, 0.4 m wide (along the slope) and 1.5 m tall, stands on a rough plane ($\mu = 0.45$) whose angle is slowly increased. Does it slide or topple first, and at what angle?

3 A uniform solid cylinder of radius 0.25 m and height 0.8 m stands on a plane face on a rough plane ($\mu = 0.6$) whose angle is slowly increased. Does it slide or topple first, and at what angle?

4 A uniform cube of side 0.5 m stands on a rough plane ($\mu = 1.2$) whose angle is slowly increased. Does it slide or topple first, and at what angle?

B · Ranges, composite bodies and solids A range: write the condition as an inequality. Find G first (Units 1–2): h is its height above the plane, d its distance from the lowest edge along the slope.

5 A uniform block, 0.5 m wide (along the slope) and H metres tall, stands on a rough plane ($\mu = 0.4$) whose angle is slowly increased. Find the range of values of H for which it slides before it topples.

6 A rough plane is fixed at $25°$ to the horizontal. A uniform block, 0.4 m wide (along the slope) and H metres tall, is placed on it ($\mu = 0.8$).
(a) Show that the block does not slide.
(b) Find the greatest value of H for which it does not topple.

7 A uniform lamina is a rectangle 0.6 m wide and 0.3 m tall with an isosceles triangle of height 0.6 m on its top edge (the triangle's base is the whole top edge). It stands on the bottom edge of the rectangle in a vertical plane containing a line of greatest slope of a rough plane ($\mu = 0.8$) whose angle is slowly increased.
(a) Show that its centre of mass is 0.325 m above its base.
(b) Does it slide or topple first, and at what angle?

8 A uniform solid cone of base radius 0.1 m and height 0.6 m stands on its base on a rough plane ($\mu = 0.7$) whose angle is slowly increased. Does it slide or topple first, and at what angle?

9 A uniform solid paraboloid (Lesson 2) has height 0.9 m and a plane face of radius 0.6 m; its centre of mass is on its axis, two-thirds of the height from the vertex. It stands on its plane face on a rough plane whose angle is slowly increased.
(a) Find the angle at which it would topple.
(b) Given $\mu = 0.9$, which happens first, and at what angle?

C · Reasoning No calculator. Explain in words.

10 Two blocks have the same size and shape, but one is made of a denser material and weighs twice as much. They stand side by side on the same rough plane (same μ), whose angle is slowly increased. A pupil says: "The heavier block will start to slide at a smaller angle." Is the pupil right? Explain.

11 A short block and a tall block have the same width and stand on the same rough plane, whose angle is slowly increased. On level ground (Lesson 3) only the horizontal distance of G from the front edge mattered for toppling. Explain why the height of G matters on a slope, and say which block topples at the smaller angle.

12 A uniform solid cylinder stands on a rough plane whose angle is slowly increased. It is replaced by a cylinder of the same radius and material but half the height. Without calculating, say what happens to
(a) the angle at which it would slide
(b) the angle at which it would topple.

D · Show that Keep letters. Write each condition as an inequality and simplify.

13 A uniform block of width $2a$ (along the slope) and height $2b$ stands on a rough plane (coefficient of friction μ) whose angle is slowly increased.
(a) Show that it topples before it slides if $b > a/\mu$.
(b) A block is 0.5 m wide and $\mu = 0.4$. Find the range of heights for which it topples first.

14 A uniform solid cone of base radius r and height H stands on its base on a rough plane (coefficient of friction μ).
(a) Show that, as the angle of the plane is slowly increased, it topples before it slides if $H > 4r/\mu$.
(b) A cone has base radius 5 cm and $\mu = 0.8$. Find the range of heights for which it topples first.

15 A box of weight W rests on a rough plane inclined at α (coefficient of friction μ). A horizontal force P, in the vertical plane through G containing a line of greatest slope, pushes the box towards the plane. The box does not topple.
(a) Show that it is about to slide up the plane when $P = W(\sin\alpha + \mu\cos\alpha)/(\cos\alpha - \mu\sin\alpha)$.
(b) Find P when $W = 100$ N, $\alpha = 20°$ and $\mu = 0.3$.

E · Exam-style question Show your method: marks are for working.

16 A uniform lamina OABC has vertices O$(0, 0)$, A$(0.9, 0)$, B$(0.9, 0.3)$ and C$(0, 0.9)$ (metres): a rectangle with a right-angled triangle on top.
(a) Show that its centre of mass is 0.375 m from OC and 0.325 m from OA. [4]
The lamina stands in a vertical plane with OA along a line of greatest slope of a rough plane. The angle of the plane is slowly increased.
(b) A is the lower end of OA. Assuming it does not slide first, find the angle at which the lamina would topple. [2]
(c) The lamina is turned round so that O is the lower end, and $\mu = 1.2$. Determine whether it slides or topples first. [2]