Moments and Couples

Specification: AQA 7367: ME6 (moments and couples; A level only) · Edexcel 9FM0 FM2: not examined

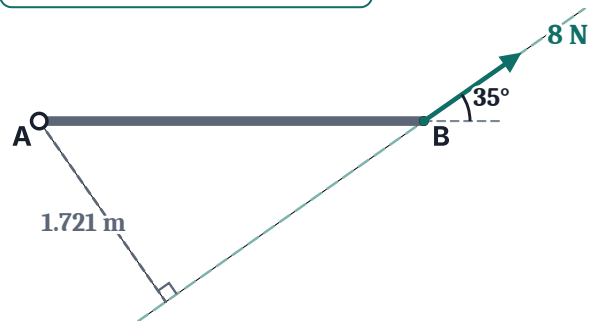
INDEPENDENT ## You do

A · Moments F × perpendicular distance; $x F_y - y F_x$.

1 A force of 8 N acts at the end B of a light rod AB of length 3 m, at 35° to the rod. Find the magnitude of its moment about A.

$d = 3 \sin 35° = 1.721$ m — perpendicular distance

Moment $= 8 \times 1.721 = 13.8$ N m — F × d



picture: d = 1.721 m

2 A force $(5\mathbf{i} + 2\mathbf{j})$ N acts at the point $(3, -1)$. Find its moment about O.

$3 \times 2 - (-1) \times 5 = 6 + 5 = 11$ — k-component of r × F

11 N m anticlockwise — sense

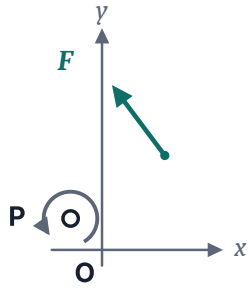


picture: anticlockwise about O

3 A force $(-3\mathbf{i} + 4\mathbf{j})$ N acts at the point $(2, 3)$. Find its moment about the point P$(-1, 1)$.

$r - p = 3\mathbf{i} + 2\mathbf{j}$ — position from P

$3 \times 4 - 2 \times (-3) = 12 + 6 = 18$: 18 N m anticlockwise — k-component of r × F

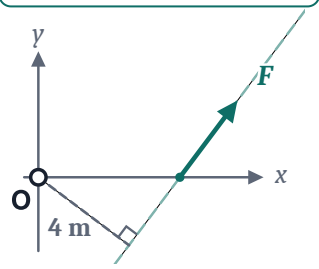


picture: anticlockwise about P

4 A force $(6\mathbf{i} + 8\mathbf{j})$ N acts at the point $(5, 0)$. Find the perpendicular distance from O to its line of action.

Moment about O: $5 \times 8 - 0 \times 6 = 40$ — moment about O

$|F| = 10$: $d = 40 \div 10 = 4$ m — d = moment ÷ F



picture: d = 4 m

B · Reducing a system $R = (\Sigma F_x, \Sigma F_y)$; line of action $x\,R_y - y\,R_x = M_O$; at A: $M_A = M_O - a \times R$.

5 A rectangular lamina OABC has O$(0,0)$, A$(6,0)$, B$(6,4)$ and C$(0,4)$. Forces of 6 N along OA, 2 N along AB and 6 N along OC act on it, in the directions given by the letters.
(a) Find the magnitude of the resultant and the angle it makes with OA.
(b) Find where its line of action crosses OA.

(a) $R = 6\mathbf{i} + 8\mathbf{j}$: $|R| = 10$ N, at $\tan^{-1}(8/6) = 53.1°$ to OA

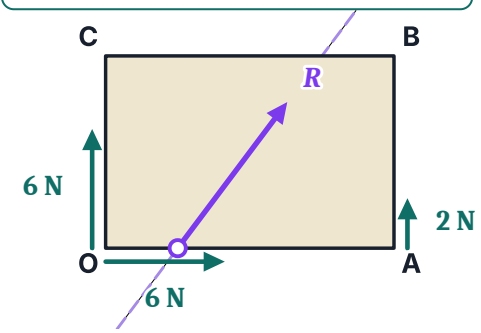
add the components

(b) $M_O = 6 \times 2 = 12$ (OA and OC pass through O)

moments about O

$8x - 6y = 12$; $y = 0$: $x = 1.5$: 1.5 m from O

put y = 0



picture: R crosses OA 1.5 m from O

6 A square lamina ABCD has side 3 m (A, B, C, D anticlockwise). Forces of 5 N along AB, 2 N along BC, 5 N along CD and 2 N along DA act on it.
(a) Show that the resultant is zero.
(b) Find the moment of the couple.

(a) AB and CD are opposite: $5 - 5 = 0$; BC and DA: $2 - 2 = 0$: $R = 0$

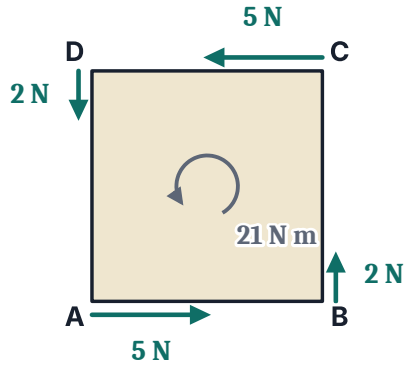
opposite pairs cancel

(b) About A: AB and DA pass through A; BC: $2 \times 3 = 6$; CD: $5 \times 3 = 15$

moments about A

Couple: $6 + 15 = 21$: 21 N m anticlockwise

add



picture: an anticlockwise couple

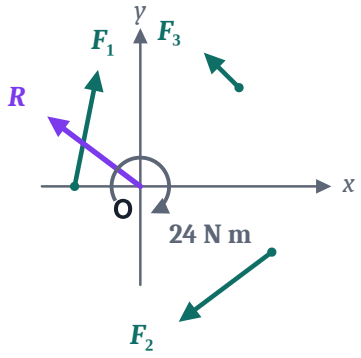
7 Forces $(\mathbf{i} + 5\mathbf{j})$ N, $(-4\mathbf{i} - 3\mathbf{j})$ N and $(-\mathbf{i} + \mathbf{j})$ N act at the points $(-2, 0)$, $(4, -2)$ and $(3, 3)$.
(a) Reduce the system to a single force at O and a couple.
(b) Find where the line of action of the single equivalent force crosses the x-axis.

(a) $R = -4\mathbf{i} + 3\mathbf{j}$

add the components

$M_O = (-10) + (-12 - 8) + (3 + 3) = -24$: $-4\mathbf{i} + 3\mathbf{j}$ at O and a couple of 24 N m clockwise

total moment about O



picture: R at O and a clockwise couple

(b) $3x + 4y = -24$; $y = 0$: $x = -8$: $(-8, 0)$

put y = 0

8 Forces $(-3\mathbf{i} + 3\mathbf{j})$ N, $(\mathbf{i} + 2\mathbf{j})$ N and $(-2\mathbf{i} - 2\mathbf{j})$ N act at the points $(3, -1)$, $(-1, 4)$ and $(4, -2)$. Reduce the system to a single force at A$(2, 1)$ and a couple.

$R = -4\mathbf{i} + 3\mathbf{j}$; $M_O = (9 - 3) + (-2 - 4) + (-8 - 4) = -12$

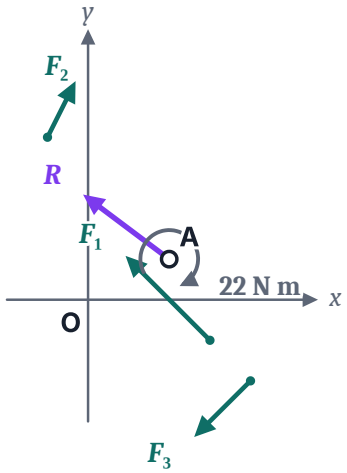
R and the moment about O

$M_A = -12 - (2 \times 3 - 1 \times (-4)) = -12 - 10 = -22$

shift the couple to A

$-4\mathbf{i} + 3\mathbf{j}$ at A and a couple of 22 N m clockwise

force + couple



picture: R at A and a clockwise couple

9 A triangular lamina has vertices O$(0,0)$, A$(4,0)$, B$(0,3)$. Forces: 4 N along AO, 5 N along AB, 3 N along OB.
(a) Find the magnitude of the resultant.
(b) Where does its line of action cross OA and OB?

(a) AB: $5 \times (-4\mathbf{i} + 3\mathbf{j})/5$. $R = (-4 - 4)\mathbf{i} + (3 + 3)\mathbf{j} = -8\mathbf{i} + 6\mathbf{j}$, $|R| = 10$ N

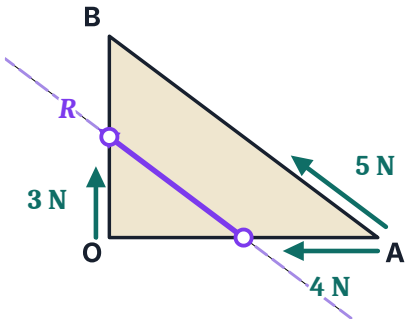
add the components

(b) $M_O = 4 \times 3 = 12$ (AB acts at A; AO and OB pass through O)

moments about O

$6x + 8y = 12$: OA at $x = 2$, OB at $y = 1.5$ (both midpoints)

put y = 0, then x = 0



picture: R through both midpoints

C · Reasoning (no calculator) Explain, or give a counter-example.

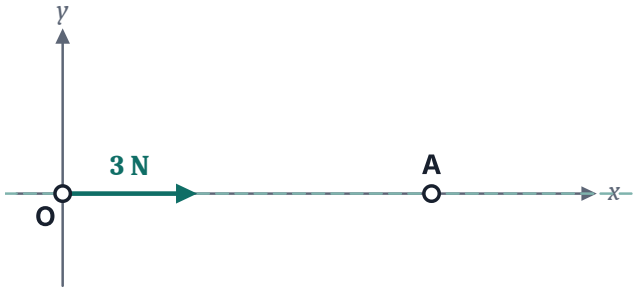
10 A system of forces has total moment zero about O and also about A$(4,0)$. Must it be in equilibrium? Explain, with an example.

No. A single force along the x-axis, e.g. $3\mathbf{i}$ acting at O, has zero moment about O and about A: its line of action passes through both.

counter-example

Equilibrium also needs $R = 0$ (or zero moment about a third point not on the line OA, the x-axis).

what else is needed



picture: the line of action passes through O and A

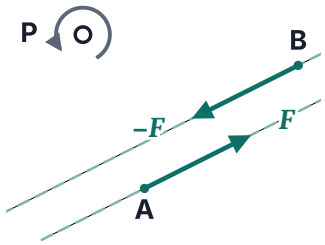
11 Forces F and $-F$ act at the points with position vectors a and b. Show that their total moment about the point with position vector p does not depend on p.

$(a - p) \times F + (b - p) \times (-F) = a \times F - p \times F - b \times F + p \times F$

expand

$= (a - b) \times F$: no p, so the same about every point.

p cancels



picture: a couple

12 Can a couple be balanced by a single force? Explain.

Equilibrium needs the forces to add to zero. A couple already adds to zero, so the single force would have to be zero: then nothing balances the couple's moment.

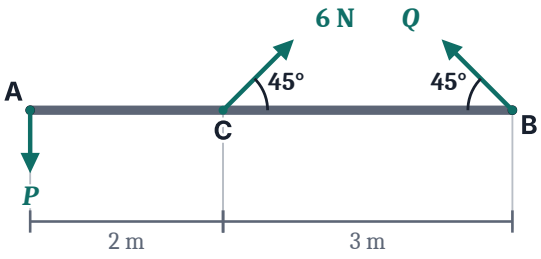
resultant must be 0

No. A couple can only be balanced by an equal and opposite couple.

only by a couple

D · Show that Resolve, then take moments. Keep surds exact.

13 A light rod AB has length 5 m; C is on AB with AC = 2 m and CB = 3 m. Forces act as shown: P newtons at A, perpendicular to the rod; 6 N at C, at 45° to the rod; Q newtons at B, at 45° to the rod. They are equivalent to a couple.
(a) Show that $Q = 6$ and $P = 6\sqrt{2}$.
(b) Find the moment of the couple.



(a) Along the rod: $6\cos 45° = Q\cos 45°$: $Q = 6$

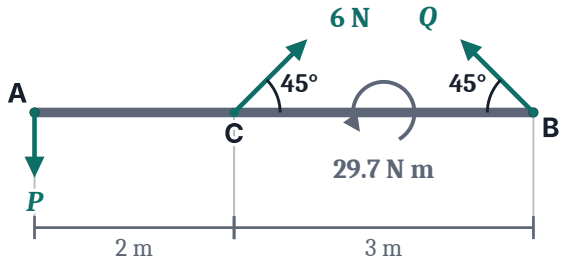
resolve along the rod

Perpendicular: $P = 6\sin 45° + 6\sin 45° = 6\sqrt{2}$

resolve ⊥ the rod

(b) About A: $2 \times 3\sqrt{2} + 5 \times 3\sqrt{2} = 21\sqrt{2} = 29.7$ N m anticlockwise

moments about A



picture: the couple, anticlockwise

14 Forces $(2\mathbf{i} + \mathbf{j})$ N, $(\mathbf{i} - 3\mathbf{j})$ N and $(k\mathbf{i} + 2\mathbf{j})$ N act at the points $(1, 0)$, $(0, 2)$ and $(3, 1)$.
(a) Show that the total moment about O is $(5 - k)$ N m.
(b) The line of action of the resultant passes through O. Find k and the resultant.

(a) $1 \times 1 - 0 = 1$; $0 \times (-3) - 2 \times 1 = -2$; $3 \times 2 - 1 \times k = 6 - k$

r × F for each

Total: $1 - 2 + 6 - k = 5 - k$

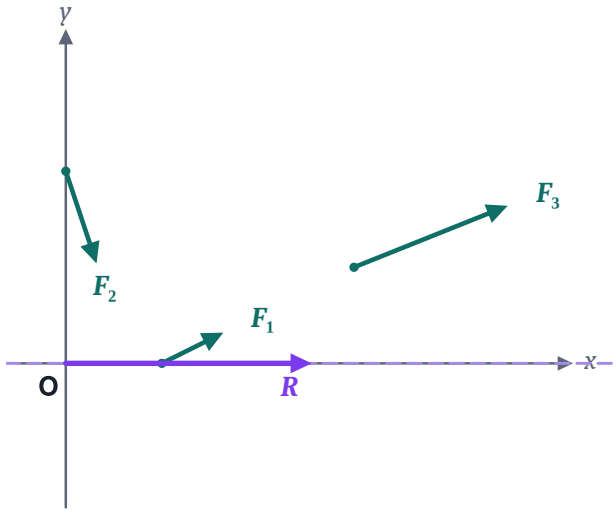
add

(b) Through O, so the moment about O is 0: $k = 5$

moment about O = 0

$R = (2 + 1 + 5)\mathbf{i} + (1 - 3 + 2)\mathbf{j} = 8\mathbf{i}$ N

add the components



picture: R = 8i through O

15 Forces $F_1 = (2\mathbf{i} - \mathbf{j} + 2\mathbf{k})$ N and $F_2 = (-\mathbf{i} + 4\mathbf{j} + \mathbf{k})$ N act at the points P$(1, 2, -1)$ and Q$(0, 1, 2)$. A third force F_3, also acting at Q, makes the three forces a couple.
(a) Find F_3 and its magnitude.
(b) Find the moment of the couple as a vector, and its magnitude.

(a) $F_3 = -(F_1 + F_2) = -\mathbf{i} - 3\mathbf{j} - 3\mathbf{k}$: $|F_3| = \sqrt{19} = 4.36$ N

resultant zero

(b) About Q (F_2 and F_3 act at Q), with position vectors p, q: $(p - q) \times F_1 = (\mathbf{i} + \mathbf{j} - 3\mathbf{k}) \times (2\mathbf{i} - \mathbf{j} + 2\mathbf{k})$

moments about Q

$= (2 - 3)\mathbf{i} - (2 + 6)\mathbf{j} + (-1 - 2)\mathbf{k} = -\mathbf{i} - 8\mathbf{j} - 3\mathbf{k}$

vector product

Magnitude $\sqrt{(1 + 64 + 9)} = \sqrt{74} = 8.60$ N m

magnitude

E · Exam-style question Show your method: marks are for working.

16 A rectangular lamina OABC has OA = 5 m and AB = 3 m (O at the origin, A on the positive x-axis, C on the positive y-axis). Forces of 4 N along OA, 6 N along AB and 4 N along CB act on it, in the directions given by the letters.

(a) Find the magnitude of the resultant force and the angle it makes with OA. [3]

(b) Find the distance from O of the point where the line of action of the resultant crosses OA. [3]

(c) The system is replaced by a single force acting at C together with a couple. Find the moment of the couple. [2]

(a) $R = (4 + 4)\mathbf{i} + 6\mathbf{j} = 8\mathbf{i} + 6\mathbf{j}$ **B1** — add the components

$|R| = \sqrt{(8^2 + 6^2)}$, angle $\tan^{-1}(6/8)$ **M1** — Pythagoras; $\tan^{-1}$

10 N at 36.9° to OA **A1** — both

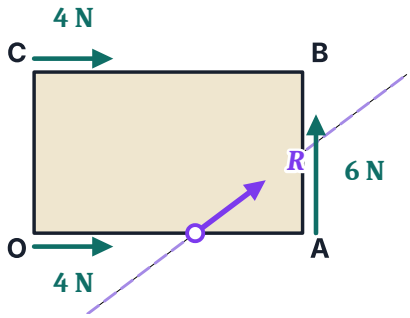
(b) About O: AB: $6 \times 5 = 30$; CB, 3 m above O: $-4 \times 3 = -12$; total $30 - 12 = 18$ **M1** — moments about O

Crossing OA at $(a, 0)$: $6a = 18$ **M1** — moment of R

$a = 3$: 3 m from O **A1** — solve

(c) $M_C = 18 - (0 \times 6 - 3 \times 8) = 18 + 24 = 42$ **M1** — shift the couple to C

42 N m anticlockwise **A1** — sense



picture: R crosses OA 3 m from O