Moments and Couples

Specification: AQA 7367: ME6 (moments and couples; A level only) · Edexcel 9FM0 FM2: not examined

INDEPENDENT

You do

A · Moments F × perpendicular distance; $x\,F_y - y\,F_x$.

1 A force of 8 N acts at the end B of a light rod AB of length 3 m, at 35° to the rod. Find the magnitude of its moment about A.

2 A force $(5\mathbf{i} + 2\mathbf{j})$ N acts at the point $(3, -1)$. Find its moment about O.

3 A force $(-3\mathbf{i} + 4\mathbf{j})$ N acts at the point $(2, 3)$. Find its moment about the point P$(-1, 1)$.

4 A force $(6\mathbf{i} + 8\mathbf{j})$ N acts at the point $(5, 0)$. Find the perpendicular distance from O to its line of action.

B · Reducing a system $R = (\Sigma F_x, \Sigma F_y)$; line of action $x\,R_y - y\,R_x = M_O$; at A: $M_A = M_O - a \times R$.

5 A rectangular lamina OABC has O$(0, 0)$, A$(6, 0)$, B$(6, 4)$ and C$(0, 4)$. Forces of 6 N along OA, 2 N along AB and 6 N along OC act on it, in the directions given by the letters.
(a) Find the magnitude of the resultant and the angle it makes with OA.
(b) Find where its line of action crosses OA.

6 A square lamina ABCD has side 3 m (A, B, C, D anticlockwise). Forces of 5 N along AB, 2 N along BC, 5 N along CD and 2 N along DA act on it.
(a) Show that the resultant is zero.
(b) Find the moment of the couple.

7 Forces $(\mathbf{i} + 5\mathbf{j})$ N, $(-4\mathbf{i} - 3\mathbf{j})$ N and $(-\mathbf{i} + \mathbf{j})$ N act at the points $(-2, 0)$, $(4, -2)$ and $(3, 3)$.
(a) Reduce the system to a single force at O and a couple.
(b) Find where the line of action of the single equivalent force crosses the x-axis.

8 Forces $(-3\mathbf{i} + 3\mathbf{j})$ N, $(\mathbf{i} + 2\mathbf{j})$ N and $(-2\mathbf{i} - 2\mathbf{j})$ N act at the points $(3, -1)$, $(-1, 4)$ and $(4, -2)$. Reduce the system to a single force at A$(2, 1)$ and a couple.

9 A triangular lamina has vertices O$(0, 0)$, A$(4, 0)$, B$(0, 3)$. Forces: 4 N along AO, 5 N along AB, 3 N along OB.
(a) Find the magnitude of the resultant.
(b) Where does its line of action cross OA and OB?

C · Reasoning (no calculator) Explain, or give a counter-example.

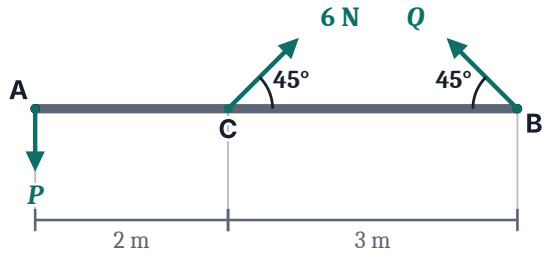
10 A system of forces has total moment zero about O and also about A$(4, 0)$. Must it be in equilibrium? Explain, with an example.

11 Forces F and $-F$ act at the points with position vectors a and b. Show that their total moment about the point with position vector p does not depend on p.

12 Can a couple be balanced by a single force? Explain.

D · Show that Resolve, then take moments. Keep surds exact.

13 A light rod AB has length 5 m; C is on AB with AC = 2 m and CB = 3 m. Forces act as shown: P newtons at A, perpendicular to the rod; 6 N at C, at 45° to the rod; Q newtons at B, at 45° to the rod. They are equivalent to a couple.
(a) Show that $Q = 6$ and $P = 6\sqrt{2}$.
(b) Find the moment of the couple.



14 Forces $(2\mathbf{i} + \mathbf{j})$ N, $(\mathbf{i} - 3\mathbf{j})$ N and $(k\mathbf{i} + 2\mathbf{j})$ N act at the points $(1, 0)$, $(0, 2)$ and $(3, 1)$.
(a) Show that the total moment about O is $(5 - k)$ N m.
(b) The line of action of the resultant passes through O. Find k and the resultant.

15 Forces $F_1 = (2\mathbf{i} - \mathbf{j} + 2\mathbf{k})$ N and $F_2 = (-\mathbf{i} + 4\mathbf{j} + \mathbf{k})$ N act at the points P$(1, 2, -1)$ and Q$(0, 1, 2)$. A third force F_3, also acting at Q, makes the three forces a couple.
(a) Find F_3 and its magnitude.
(b) Find the moment of the couple as a vector, and its magnitude.

E · Exam-style question Show your method: marks are for working.

16 A rectangular lamina OABC has OA = 5 m and AB = 3 m (O at the origin, A on the positive x-axis, C on the positive y-axis). Forces of 4 N along OA, 6 N along AB and 4 N along CB act on it, in the directions given by the letters.
(a) Find the magnitude of the resultant force and the angle it makes with OA. [3]
(b) Find the distance from O of the point where the line of action of the resultant crosses OA. [3]
(c) The system is replaced by a single force acting at C together with a couple. Find the moment of the couple. [2]