Rigid Bodies in Equilibrium

Specification: AQA 7367: ME6 (rigid bodies in equilibrium; A level only) · Edexcel 9FM0 FM2: not examined

INDEPENDENT

You do

A · Ladders and pegs Smooth wall: $R = W$, $F = S$, moments about the foot. Peg: reaction $\perp$ the rod.

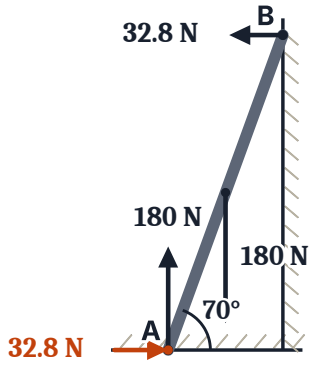
1 A uniform ladder of length 6 m and weight 180 N rests with its foot on rough horizontal ground and its top against a smooth vertical wall, at 70° to the ground. Find the reaction at the wall, and the normal reaction and the friction at the foot.

About the foot: $S \times 6 \sin 70° = 180 \times 3 \cos 70°$, $S = 32.8$ N

moments about the foot

$R = 180$ N, $F = S = 32.8$ N

resolve twice



picture: the forces

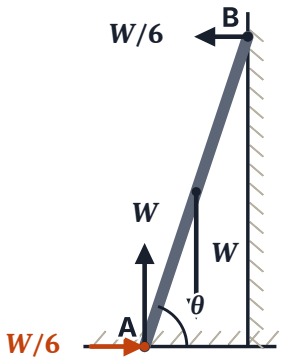
2 A uniform ladder rests against a smooth vertical wall with its foot on rough horizontal ground, at an angle θ to the ground where $\tan\theta = 3$. Find the least possible coefficient of friction.

From the derivation: $S = W/(2\tan\theta) = W/6$, $R = W$

S = W/(2 tan θ)

$\mu \geq (W/6) \div W = 1/6 = 0.167$

F ≤ μR



picture: on the point of slipping

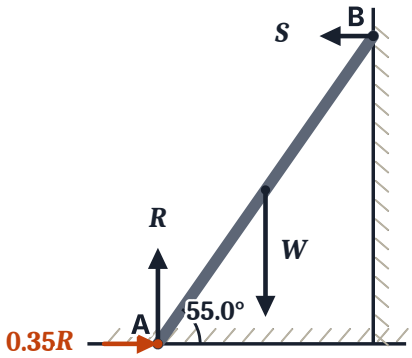
3 A uniform ladder rests against a smooth vertical wall with its foot on rough horizontal ground ($\mu = 0.35$). Find the least angle it can make with the ground.

$\tan\theta = 1/(2 \times 0.35) = 1.429$

tan θ = 1/(2μ)

$\theta = 55.0°$

inverse tan



picture: at the least angle

4 A uniform rod AB, of length 2 m and weight 30 N, has A on rough horizontal ground and rests on a smooth peg at C, where AC = 1.5 m, at 45° to the horizontal. Find the reaction at the peg and the least possible value of μ.

About A: $N \times 1.5 = 30 \times 1 \times \cos 45°$, $N = 10\sqrt{2} = 14.1$ N

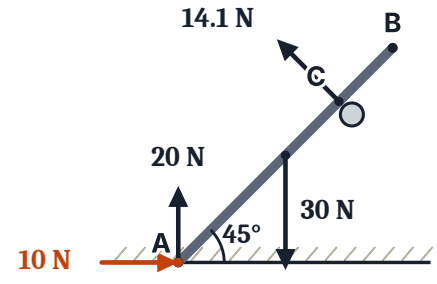
moments about A

$F = N \sin 45° = 10$ N, $R = 30 - N\cos 45° = 20$ N

resolve twice

$\mu \geq 10 \div 20 = 0.5$

F ≤ μR



picture: N ⊥ the rod

B · Climbers, two rough contacts, bowls and strings

About to slip: $F = \mu R$ at every rough contact. Three forces meet at a point.

5 The ladder of Do Now Q3 (length 5 m, mass 20 kg, a man of mass 80 kg at C with AC = 4 m) rests with A on rough horizontal ground and B against a smooth vertical wall, at θ to the ground where $\tan\theta = 2$. Find the least possible value of μ.

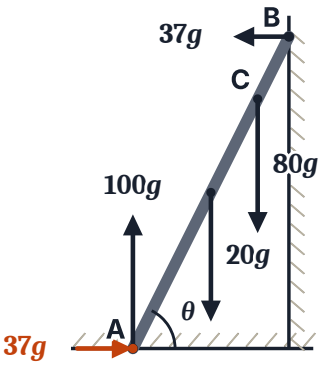
The combined weight $100g$ acts 3.7 m from A (Do Now Q3). About A: $S \times 5\sin\theta = 100g \times 3.7\cos\theta$

moments about A ÷ cos θ; resolve

$S = 370g \div (5 \times 2) = 37g$; $R = 100g$

$\mu \geq 37g \div 100g = 0.37$

F ≤ μR



picture: the forces

6 A uniform ladder AB, of length $2a$, rests with A on rough horizontal ground (μ = 0.3) and B against a rough vertical wall (μ′ = 0.6). Find the least angle it can make with the ground.

About to slip: $S = 0.3R$, $R + 0.6S = W$: $1.18R = W$

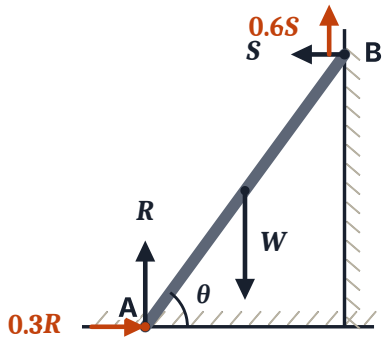
both limiting; resolve

About A (length $2a$): $2S\tan\theta + 1.2S = W$, $W/S = 1.18/0.3 = 3.933$

moments about A

$\tan\theta = (1.18/0.3 - 1.2) \div 2 = 1.367$, $\theta = 53.8°$

solve



picture: limiting friction at both ends

7 A smooth hemispherical bowl has centre O and radius 0.3 m, rim horizontal. A uniform rod AB of weight 20 N rests with A on the inside of the bowl, touching the rim at C, at 25° to the horizontal.
(a) Use the fact that the three forces meet at a point to find the length of the rod.
(b) Find the reaction at A.

(a) The reaction at A acts along AO; the reaction at C ⊥ the rod. They meet at P on the circle that completes the bowl's cross-section, with AP a diameter (angle ACP = 90°).

lines of action

The weight passes through P too, so G is vertically below P: $(l/2)\cos 25° = 2 \times 0.3 \cos 50°$

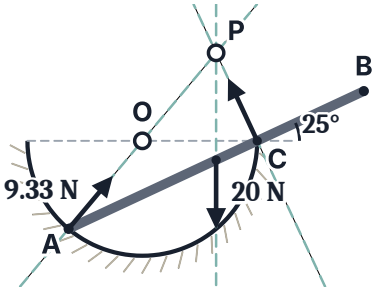
G below P

$l = 4 \times 0.3 \cos 50° \div \cos 25° = 0.851$ m

solve

(b) Along the rod: $R_A \cos 25° = 20 \sin 25°$, $R_A = 20 \tan 25° = 9.33$ N

resolve along the rod



picture: the three forces meet at P

8 A uniform rectangular lamina ABCD, with AB = 0.8 m and BC = 0.6 m, has weight 50 N. It is freely hinged at A to a fixed point and held in a vertical plane, with AB horizontal and D vertically above A, by a light string from C to the point E, 1.2 m vertically above A.
(a) Find the tension in the string.
(b) Find the magnitude and direction of the force at the hinge.

(a) With A as the origin: C(0.8, 0.6), E(0, 1.2), CE = (−0.8, 0.6), length 1: $T = (-0.8T, 0.6T)$. G(0.4, 0.3)

unit vector along CE

About A: $0.8 \times 0.6T - 0.6 \times (-0.8T) - 50 \times 0.4 = 0$: $0.96T = 20$, $T = 20.8$ N

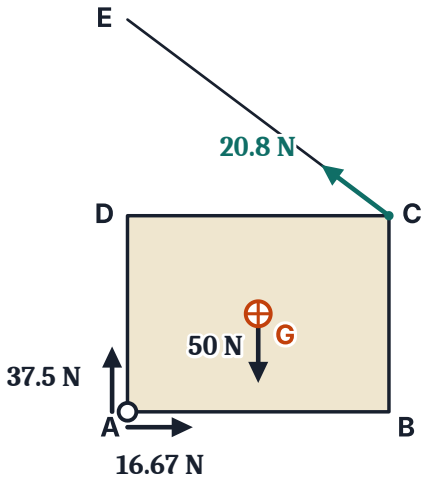
moments about A

(b) →: $X = 0.8T = 16.67$; ↑: $Y = 50 - 0.6T = 37.5$

resolve twice

$\sqrt{(16.67^2 + 37.5^2)} = 41.0$ N at $\tan^{-1}(37.5/16.67) = 66.0°$ above the horizontal

magnitude and direction



picture: the forces on the lamina

9 A uniform rod AB, of length 2 m and weight 40 N, is freely hinged at A to a vertical wall. It is held horizontal by a light string from B to the point D on the wall, 1.5 m vertically above A. Use the fact that the three forces meet at a point to find the direction of the force at the hinge, then find its magnitude and the tension.

Above the midpoint M the string is at half the height of D, 0.75 m: the string and the weight meet there.

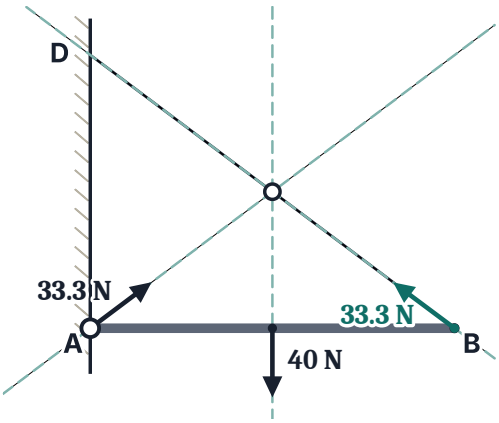
lines meet

So the hinge force acts along the line from A to that point: $\tan^{-1}(0.75/1) = 36.9°$ above the horizontal.

hinge force through it

The string also makes $36.9°$ with the rod. →:
$H\cos 36.9° = T\cos 36.9°$, $H = T$; ↑: $2T \times 0.6 = 40$

resolve twice



picture: the three lines meet

$T = H = 33.3$ N

solve

C · Reasoning (no calculator) Explain in words, with a moment or a resolution where it helps.

10 A ladder rests with its foot on **smooth** horizontal ground and its top against a **rough** vertical wall. Explain why it cannot be in equilibrium (unless it is vertical).

The only horizontal forces are the wall's normal reaction S (the ground is smooth). Resolving horizontally: $S = 0$.

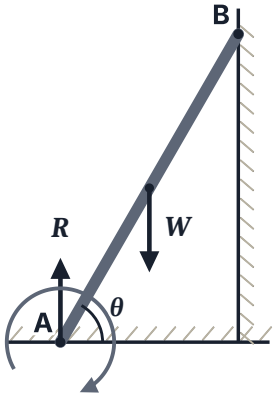
resolve horizontally

Then the wall's friction is at most $\mu' S = 0$. Only R and W are left, both vertical but along different lines.

friction ≤ μ'S = 0

About A, W has a moment and nothing balances it, so the ladder slips.

moments about A



picture: W has an unbalanced moment

11 A uniform ladder against a smooth wall has least angle given by $\tan\theta = 1/(2\mu)$.
(a) Explain why a heavier (or longer) ladder has the same least angle.
(b) Find the least angle when $\mu = 0.5$.

(a) Every force is proportional to W ($R = W$, $S = W/(2\tan\theta)$), so W cancels in $F \le \mu R$; the length cancels in the moments.

W and length cancel

(b) $\tan\theta = 1/(2 \times 0.5) = 1$: $\theta = 45°$

tan θ = 1/(2μ)

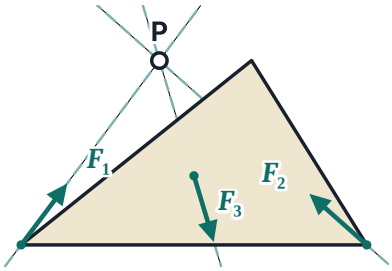
12 Three non-parallel forces act on a rigid body in equilibrium. The lines of action of two of them meet at the point P. Explain why the third force must also pass through P.

Take moments about P: the first two forces have no moment about P.

moments about P

The total moment is zero, so the third force has zero moment about P: its line of action passes through P.

moment = 0 ⇒ through P



picture: the three lines meet at P

D · Algebra and "show that"

Keep W, a, μ as letters until the end.

13 A uniform ladder AB, of length $2a$ and weight W, rests with A on rough horizontal ground (coefficient μ) and B against a rough vertical wall (coefficient μ'). Show that the least angle θ with the ground satisfies $\tan\theta = (1 - \mu\mu')/(2\mu)$.

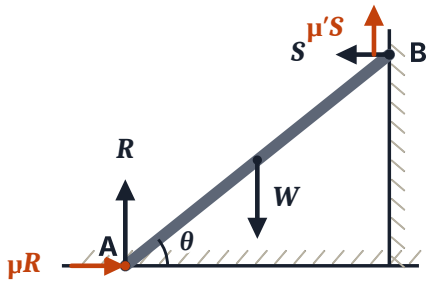
About to slip: $S = \mu R$, $R + \mu' S = W$: $R = W/(1 + \mu\mu')$, $S = \mu W/(1 + \mu\mu')$

both limiting; resolve

About A (length $2a$):
$S \times 2a \sin\theta + \mu' S \times 2a \cos\theta = W \times a \cos\theta$:
$2S \tan\theta + 2\mu' S = W$

moments about A

$\tan\theta = (W - 2\mu' S)/(2S) = (1 + \mu\mu' - 2\mu\mu')/(2\mu)$ substitute S



picture: μR and μ'S

$\tan\theta = (1 - \mu\mu')/(2\mu)$ simplify

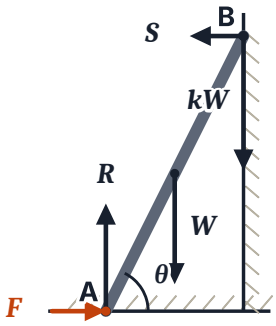
14 A uniform ladder AB, of length $2a$ and weight W, rests with A on rough horizontal ground and B against a smooth vertical wall, at θ to the ground. A man of weight kW stands at the top.
(a) Show that equilibrium needs $\mu \geq (1 + 2k)/(2(1 + k)\tan\theta)$.
(b) Find the least μ when $\tan\theta = 2$ and $k = 3$.

(a) ↑: $R = (1 + k)W$; about A:
$S \times 2a \sin\theta = Wa \cos\theta + kW \times 2a \cos\theta$

resolve; moments about A

$S = (1 + 2k)W/(2\tan\theta)$; $F = S \leq \mu R$ gives the result.

F = S ≤ μR



picture: kW at the top

(b) $\mu \geq 7/(2 \times 4 \times 2) = 7/16 = 0.4375$ substitute

15 A uniform rod of length l and weight W rests in a smooth hemispherical bowl of radius a, with one end A on the inside of the bowl and touching the rim at C, at θ to the horizontal.

(a) Show that AC = $2a\cos\theta$.

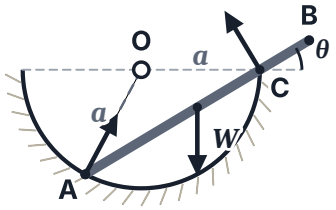
(b) Show that the reaction at C is $Wl/(4a)$, whatever the value of θ.

(a) OA = OC = a and OC is horizontal, so the triangle OAC is isosceles with base angles θ: $AC = 2a\cos\theta$

isosceles triangle

(b) About A (the reaction at A passes through A):
$R_C \times 2a\cos\theta = W \times (l/2)\cos\theta$

moments about A



picture: OA = OC = a

$R_C = Wl/(4a)$: cos θ cancels.

cos θ cancels

E · Exam-style question Show your method: marks are for working.

16 A uniform ladder AB, of length 4 m and mass 15 kg, rests with A on rough horizontal ground. It rests against the top C of a smooth vertical wall, where AC = 3 m, at 55° to the ground. A man of mass 75 kg stands on the ladder at D, where AD = 2.5 m. The ladder is on the point of slipping. Take g = 9.8 m s^{-2}.

(a) Explain why the reaction at C is perpendicular to the ladder. [1]

(b) Find the reaction at C. [3]

(c) Find the coefficient of friction between the ladder and the ground. [4]

(a) The top of the wall is smooth and the contact is with the ladder's surface, so the reaction is normal to the ladder. B1

smooth: normal to the ladder

(b) About A: $N \times 3 = 15g \times 2\cos 55° + 75g \times 2.5\cos 55°$ M1

moments about A

$3N = 217.5g\cos 55° = 1222.6$ A1

evaluate

N = 408 N A1

solve

(c) →: $F = N\sin 55° = 333.8$ M1

resolve horizontally

↑: $R = 90g - N\cos 55° = 882 - 233.7 = 648.3$ M1

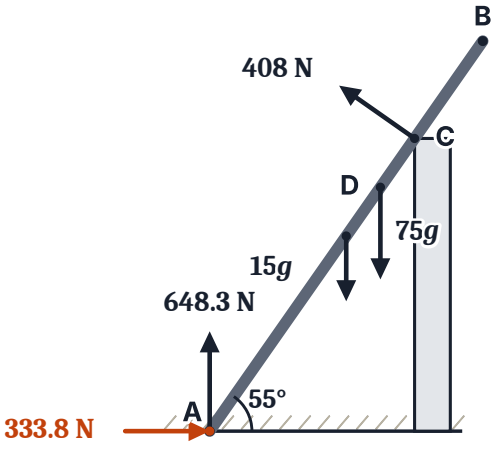
resolve vertically

$F = \mu R$: $\mu = 333.8 \div 648.3$ M1

F = μR

μ = 0.515 A1

solve



picture: N ⊥ the ladder at C