Rigid Bodies in Equilibrium

Specification: AQA 7367: ME6 (rigid bodies in equilibrium; A level only) · Edexcel 9FM0 FM2: not examined

INDEPENDENT

You do

A · Ladders and pegs Smooth wall: $R = W$, $F = S$, moments about the foot. Peg: reaction $\perp$ the rod.

1 A uniform ladder of length 6 m and weight 180 N rests with its foot on rough horizontal ground and its top against a smooth vertical wall, at 70° to the ground. Find the reaction at the wall, and the normal reaction and the friction at the foot.

2 A uniform ladder rests against a smooth vertical wall with its foot on rough horizontal ground, at an angle θ to the ground where $\tan\theta = 3$. Find the least possible coefficient of friction.

3 A uniform ladder rests against a smooth vertical wall with its foot on rough horizontal ground ($\mu = 0.35$). Find the least angle it can make with the ground.

4 A uniform rod AB, of length 2 m and weight 30 N, has A on rough horizontal ground and rests on a smooth peg at C, where AC = 1.5 m, at 45° to the horizontal. Find the reaction at the peg and the least possible value of μ.

B · Climbers, two rough contacts, bowls and strings About to slip: $F = \mu R$ at every rough contact. Three forces meet at a point.

5 The ladder of Do Now Q3 (length 5 m, mass 20 kg, a man of mass 80 kg at C with AC = 4 m) rests with A on rough horizontal ground and B against a smooth vertical wall, at θ to the ground where $\tan\theta = 2$. Find the least possible value of μ.

6 A uniform ladder AB, of length $2a$, rests with A on rough horizontal ground ($\mu = 0.3$) and B against a rough vertical wall ($\mu' = 0.6$). Find the least angle it can make with the ground.

7 A smooth hemispherical bowl has centre O and radius 0.3 m, rim horizontal. A uniform rod AB of weight 20 N rests with A on the inside of the bowl, touching the rim at C, at 25° to the horizontal.
(a) Use the fact that the three forces meet at a point to find the length of the rod.
(b) Find the reaction at A.

8 A uniform rectangular lamina ABCD, with AB = 0.8 m and BC = 0.6 m, has weight 50 N. It is freely hinged at A to a fixed point and held in a vertical plane, with AB horizontal and D vertically above A, by a light string from C to the point E, 1.2 m vertically above A.
(a) Find the tension in the string.
(b) Find the magnitude and direction of the force at the hinge.

9 A uniform rod AB, of length 2 m and weight 40 N, is freely hinged at A to a vertical wall. It is held horizontal by a light string from B to the point D on the wall, 1.5 m vertically above A. Use the fact that the three forces meet at a point to find the direction of the force at the hinge, then find its magnitude and the tension.

C · Reasoning (no calculator) Explain in words, with a moment or a resolution where it helps.

10 A ladder rests with its foot on **smooth** horizontal ground and its top against a **rough** vertical wall. Explain why it cannot be in equilibrium (unless it is vertical).

11 A uniform ladder against a smooth wall has least angle given by $\tan\theta = 1/(2\mu)$.
(a) Explain why a heavier (or longer) ladder has the same least angle.
(b) Find the least angle when $\mu = 0.5$.

12 Three non-parallel forces act on a rigid body in equilibrium. The lines of action of two of them meet at the point P. Explain why the third force must also pass through P.

D · Algebra and "show that" Keep W, a, μ as letters until the end.

13 A uniform ladder AB, of length $2a$ and weight W, rests with A on rough horizontal ground (coefficient μ) and B against a rough vertical wall (coefficient μ'). Show that the least angle θ with the ground satisfies $\tan\theta = (1 - \mu\mu')/(2\mu)$.

14 A uniform ladder AB, of length $2a$ and weight W, rests with A on rough horizontal ground and B against a smooth vertical wall, at θ to the ground. A man of weight kW stands at the top.
(a) Show that equilibrium needs $\mu \geq (1 + 2k)/(2(1 + k)\tan\theta)$.
(b) Find the least μ when $\tan\theta = 2$ and $k = 3$.

15 A uniform rod of length l and weight W rests in a smooth hemispherical bowl of radius a, with one end A on the inside of the bowl and touching the rim at C, at θ to the horizontal.

(a) Show that AC = $2a\cos\theta$.

(b) Show that the reaction at C is $Wl/(4a)$, whatever the value of θ.

E · Exam-style question Show your method: marks are for working.

16 A uniform ladder AB, of length 4 m and mass 15 kg, rests with A on rough horizontal ground. It rests against the top C of a smooth vertical wall, where AC = 3 m, at $55°$ to the ground. A man of mass 75 kg stands on the ladder at D, where AD = 2.5 m. The ladder is on the point of slipping. Take $g = 9.8$ m s^{-2}.

(a) Explain why the reaction at C is perpendicular to the ladder. [1]

(b) Find the reaction at C. [3]

(c) Find the coefficient of friction between the ladder and the ground. [4]