Unit Review

Specification: AQA 7367: ME3, ME4, ME5, ME6 · Edexcel 9FM0: FM2 Further centres of mass (ME6 content, moments and rigid bodies, is AQA only)

HOMEWORK Consolidation

1 A uniform lamina occupies the region between the line $y = x$ and the curve $y = \frac{1}{4}x^3$ for $x \geq 0$. Find its centre of mass.

AQA 7367 ME3 · Edexcel FM2 (Further centres of mass: lamina by integration)

2 A non-uniform rod AB has length 3 m. At a distance x metres from A its mass per unit length is $k(4 - x)$ kg m^{-1}, where k is a constant. The mass of the rod is 15 kg.

(a) Find k.

(b) Find the distance of its centre of mass from A.

Edexcel FM2 (Further centres of mass: non-uniform rod) · not in AQA 7367

3 The region bounded by the curve $y = \sqrt{(9 - 3x)}$ and the coordinate axes is rotated through $360°$ about the x-axis to form a uniform solid paraboloid with its plane face at O. Lengths are in centimetres.

(a) Show that its centre of mass is 1 cm from O.

(b) A uniform solid cylinder of radius 3 cm and length 2 cm, of the same material, is joined to the plane face so that the axes coincide. Find the distance of the centre of mass of the combined solid from O, and say on which side of O it lies.

AQA 7367 ME4 · Edexcel FM2 (Further centres of mass: solid of revolution)

4 A uniform block, 0.7 m wide and 1.2 m tall, of weight 140 N, stands on rough horizontal ground ($\mu = 0.4$). A force P pushes at the middle of the top edge of its back face, in the vertical plane through G, at $20°$ **below** the horizontal, towards the front edge E. Does the block slide or topple first, and at what value of P?

AQA 7367 ME5 · Edexcel FM2 (sliding or toppling: applied force)

5 A uniform solid is a cylinder of radius 0.25 m and height 0.4 m with a cone of base radius 0.25 m and height 0.6 m fixed to its top face. It stands on its bottom face on a rough plane ($\mu = 0.9$) whose angle is slowly increased.

(a) Find the height of its centre of mass above the base.

(b) Does it slide or topple first, and at what angle?

AQA 7367 ME5 · Edexcel FM2 (sliding or toppling on an inclined plane)

6 A rectangular lamina OABC has OA = 6 m and AB = 4 m, with O at the origin, A on the positive x-axis and C on the positive y-axis. Forces of 2 N along OA, 5 N along AB and 10 N along CB act on it, in the directions given by the letters.

(a) Find the magnitude of the resultant and the point where its line of action crosses OC.

(b) The system is replaced by a single force at O and a couple. Find the force and the couple.

AQA 7367 ME6 (moments, couples, reducing a system) · Edexcel FM2: not examined

7 Forces $(4\mathbf{i} - \mathbf{j})$ N, $(-5\mathbf{i} + 3\mathbf{j})$ N and $(p\mathbf{i} + q\mathbf{j})$ N act at the points $(1, 2)$, $(-2, 0)$ and $(1, 3)$ (metres). The system reduces to a couple.

(a) Find p and q.

(b) Find the moment of the couple.

AQA 7367 ME6 (moments, couples, reducing a system) · Edexcel FM2: not examined

8 A ladder AB, of length 5 m and mass 25 kg, is **not** uniform: its centre of mass is 2 m from A. It rests with A on rough horizontal ground ($\mu = 0.35$) and B against a smooth vertical wall, at $65°$ to the ground. A man of mass 75 kg climbs the ladder. How far from A can he go before the ladder slips? Take $g = 9.8$ m s^{-2}.

AQA 7367 ME6 (rigid bodies in equilibrium) · Edexcel FM2: not examined

9 A uniform rod AB, of length 2 m and weight 80 N, is freely hinged at A to a vertical wall. It is held at $30°$ above the horizontal by a light horizontal string attached at B and to a point of the wall vertically above A.

(a) Find the tension in the string.

(b) Find the magnitude and direction of the force at the hinge.

(c) Show that the lines of action of the three forces meet at one point.

AQA 7367 ME6 (rigid bodies in equilibrium) · Edexcel FM2: not examined

10 A uniform block, 0.7 m wide (along the slope) and 0.9 m tall, of weight 120 N, rests on a rough plane inclined at $35°$ ($\mu = 0.4$).

(a) Show that, on its own, it would slide but would not topple.

(b) A force P, parallel to the plane and up the slope, acts at the middle of its downhill face, in the vertical plane through G. Find the least value of P that stops it sliding down.

AQA 7367 ME5 · Edexcel FM2 (sliding or toppling on an inclined plane)

11 *AQA-style*. A uniform rectangular lamina OABC, with O$(0,0)$, A$(4,0)$, B$(4,2)$ and C$(0,2)$ (metres), has weight 60 N. It stands in a vertical plane with OA on rough horizontal ground. A force $(20\mathbf{i} + 10\mathbf{j})$ N acts on the lamina at C.

(a) Find the moment of this force about A, stating its sense. [2]

(b) Show that the lamina does not turn about A. [2]

(c) The lamina is about to slide. Find the coefficient of friction. [3]

AQA 7367 ME5, ME6 (toppling; moment as r × F) · Edexcel FM2: not examined (vector moments) · AQA-style

12 *Edexcel-style*. A uniform lamina OAB is bounded by the curve $y = \frac{1}{4}x^2$, the x-axis and the line $x = 4$, with O$(0,0)$, A$(4,0)$ and B$(4,4)$ (metres).

(a) Show that the centre of mass of the lamina is at $(3, 1.2)$. [3]

The lamina stands in a vertical plane with OA along a line of greatest slope of a rough plane, A below O. The angle of the plane is slowly increased.

(b) Assuming that the lamina does not slide first, find the angle at which it topples. [2]

(c) The coefficient of friction is 0.9. Does the lamina slide or topple first? [1]

AQA 7367 ME5 · Edexcel FM2 (sliding or toppling on an inclined plane) · Edexcel-style