Unit Review

Specification: AQA 7367: ME3, ME4, ME5, ME6 · Edexcel 9FM0: FM2 Further centres of mass (ME6 content, moments and rigid bodies, is AQA only)

INDEPENDENT

Mixed practice

A · Fluency One tool each. Which one?

1 A uniform lamina is bounded by the curve $y = 5x^4$, the x-axis and the line $x = 1$. Find its centre of mass.

2 The region bounded by the curve $y = x^3$, the x-axis and the line $x = 1$ is rotated through $360°$ about the x-axis to form a uniform solid. Find $\bar{x}$.

3 A uniform block, 0.9 m wide and 1.5 m tall, of weight 120 N, stands on rough horizontal ground ($\mu = 0.35$). A horizontal force P acts at the middle of the top edge of its back face, at right angles to that face. Does it slide or topple first, and at what value of P?

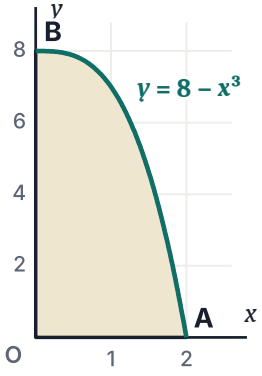
4 A uniform solid cylinder of radius 0.25 m and height 0.9 m stands on a plane face on a rough plane ($\mu = 0.6$) whose angle is slowly increased. Does it slide or topple first, and at what angle?

5 A force $(-5\mathbf{i} + 12\mathbf{j})$ N acts at the point $(3, 2)$ (metres).
(a) Find its moment about O.
(b) Find the perpendicular distance from O to its line of action.

6 A uniform ladder AB, of length 4 m and mass 18 kg, rests with A on rough horizontal ground and B against a smooth vertical wall, at $60°$ to the ground. A child of mass 30 kg stands on the ladder 3 m from A. Find the least possible coefficient of friction at the ground. Take $g = 9.8$ m s^{-2}.

B · Multi-step Two or more tools in one question. Draw it first.

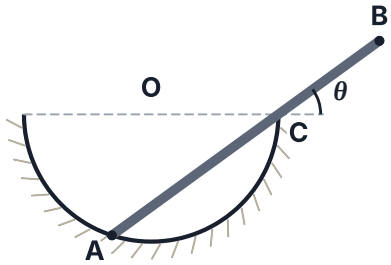
7 A uniform lamina OAB is bounded by the curve $y = 8 - x^3$ and the coordinate axes, with O at the origin, A$(2, 0)$ and B$(0, 8)$ (metres).
(a) Find its centre of mass.
The lamina stands in a vertical plane with OA along a line of greatest slope of a rough plane ($\mu = 0.3$), whose angle is slowly increased.
(b) A is the lower end of OA. Does it slide or topple first, and at what angle?
(c) The lamina is turned round so that O is the lower end. Does it slide or topple first, and at what angle?



8 A uniform lamina OABC is bounded by the curve $y = 2 + \frac{1}{2}x^2$, the coordinate axes and the line $x = 2$, with O$(0, 0)$, A$(2, 0)$, B$(2, 4)$ and C$(0, 2)$ (metres). It has weight W and stands in a vertical plane with OA on rough horizontal ground ($\mu = 0.25$).
(a) Show that $\bar{x} = 9/8$.
(b) A horizontal force P acts at B in the direction OA, and is slowly increased. Does the lamina slide or topple first?
(c) Instead, a horizontal force acts at B in the direction AO. Does it slide or topple first?

9 A square lamina OABC has side 4 m, with O at the origin, A on the positive x-axis and C on the positive y-axis. Forces of 3 N along AO, 5 N along BC, 6 N along OC and 2 N along AB act on it, in the directions given by the letters.
(a) Find the magnitude of the resultant and where its line of action crosses OA.
(b) A single force is added at C so that the system reduces to a couple. Find this force and the moment of the couple.

10 A smooth hemispherical bowl, of radius 0.5 m and centre O, is fixed with its rim horizontal. A uniform rod AB, of length 1.3 m and weight 36 N, rests with A on the inside of the bowl and the rod touching the rim at C, at θ to the horizontal.
(a) Show that $AC = \cos\theta$.
(b) Find θ.
(c) Find the reactions at A and at C.



11 A uniform block, 0.6 m wide and 1.0 m tall, of weight 200 N, stands on rough horizontal ground ($\mu = 0.3$).
(a) A couple acts on the block in the vertical plane through G that is perpendicular to its front face. Explain why the block cannot slide, and find the greatest couple for which it does not tip.
(b) Instead, a couple of 30 N m acts in the same plane, in the sense that would tip the block about its front edge E, and a horizontal force P acts at the middle of the top edge of its back face, towards E. P is slowly increased. Does the block slide or topple first, and at what value of P?

C · Reasoning No calculator. Explain in words.

12 The region bounded by $y = x^2$, the x-axis and the line $x = 1$ is used twice: as a uniform lamina, and rotated about the x-axis to make a uniform solid. Sam says: "They are made from the same region, so they have the same $\bar{x}$." Show that Sam is wrong and explain why.

13 Two uniform blocks have the same size and shape, but one weighs twice as much as the other. Each stands on the same rough horizontal ground, and a horizontal force acts on each at the same height, at the middle of its back face. Priya says: "The heavier block is harder to push, so it is more likely to topple before it slides." Is she right? Explain.

14 A rigid body is in equilibrium under three forces, two of which are parallel. Explain why the third force must be parallel to them too.

D · Show that Keep letters; simplify at the end.

15 The region bounded by the curve $y = r\sqrt{(x/h)}$, the x-axis and the line $x = h$ is rotated through $360°$ about the x-axis to form a uniform solid paraboloid with vertex O, height h and base radius r.
(a) Show that its centre of mass is $\dfrac{2h}{3}$ from O.
(b) It stands on its plane face on a rough plane (coefficient of friction μ) whose angle is slowly increased. Show that it topples before it slides if $h > 3r/\mu$.
(c) Given $r = 0.2$ m and $\mu = 0.5$, find the range of heights for which it topples first.

16 A uniform ladder AB, of length $2a$ and weight W, rests with A on rough horizontal ground (coefficient of friction μ) and B against a smooth vertical wall, at angle θ to the ground, where $\tan\theta < 1/(2\mu)$. A horizontal force P acts at A, towards the wall.
(a) Show that the least value of P for equilibrium is $\dfrac{W(1 - 2\mu\tan\theta)}{2\tan\theta}$.
(b) Find this least force when $W = 300$ N, $\theta = 50°$ and $\mu = 0.3$.

E · Exam-style questions Show your method: marks are for working.

17 *AQA-style*. A uniform lamina is bounded by the curve $y = x^2(3 - x)$ and the x-axis, between O$(0, 0)$ and A$(3, 0)$ (metres). Its weight is 60 N.

(a) Show that the centre of mass of the lamina is 1.8 m from the y-axis. [3]

The lamina is freely hinged at O to a fixed point and rests in a vertical plane with OA horizontal and the lamina above OA. It is held by a light string attached at A and to the fixed point D, 4 m vertically above the point $(6, 0)$.

(b) Find the tension in the string. [3]

(c) Find the magnitude and direction of the force at the hinge. [2]

18 *Edexcel-style*. The region bounded by the curve $y = \sqrt[3]{x}$, the x-axis and the line $x = 8$ is rotated through $360°$ about the x-axis to form a uniform solid S. Lengths are in centimetres.

(a) Show that the centre of mass of S is 5 cm from O. [4]

S stands with its plane face on a rough plane whose angle is slowly increased. The coefficient of friction is 0.6.

(b) Determine whether S slides or topples first, and find the angle at which this happens. [3]

(c) Find the range of values of the coefficient of friction for which S would topple first. [1]