The Scalar Product

Specification: AQA 7366/7367: F3, F4 · Edexcel 9FM0: Core Pure 6.3, 6.4

INDEPENDENT

You do

A · Scalar products

Multiply matching components, then add.

1 Find $\begin{pmatrix}3\\1\\4\end{pmatrix}\cdot\begin{pmatrix}2\\-5\\1\end{pmatrix}$.

2 Find $\begin{pmatrix}-2\\4\\3\end{pmatrix}\cdot\begin{pmatrix}5\\1\\-2\end{pmatrix}$.

3 Find $(2\mathbf{i}-3\mathbf{j}+\mathbf{k})\cdot(4\mathbf{i}+\mathbf{j}-2\mathbf{k})$.

4 Find $(5\mathbf{i}-2\mathbf{k})\cdot(3\mathbf{i}+7\mathbf{j}+4\mathbf{k})$.

B · Angles between vectors

$\cos\theta = \mathbf{a}\cdot\mathbf{b} \div |\mathbf{a}||\mathbf{b}|$

5 Find the angle between $\mathbf{a}=\begin{pmatrix}1\\2\\2\end{pmatrix}$ and $\mathbf{b}=\begin{pmatrix}2\\-1\\2\end{pmatrix}$.

6 Find the angle between $\mathbf{a}=\begin{pmatrix}3\\0\\4\end{pmatrix}$ and $\mathbf{b}=\begin{pmatrix}0\\5\\12\end{pmatrix}$.

7 Find the angle between $\mathbf{a}=\begin{pmatrix}2\\-3\\6\end{pmatrix}$ and $\mathbf{b}=\begin{pmatrix}-1\\4\\8\end{pmatrix}$.

8 Find the angle between $\mathbf{a}=4\mathbf{i}-\mathbf{j}+\mathbf{k}$ and $\mathbf{b}=-2\mathbf{i}+3\mathbf{j}+\mathbf{k}$.

C · Reasoning

No calculator. Explain in words as well as numbers.

9 Determine whether $\begin{pmatrix}6\\-2\\5\end{pmatrix}$ and $\begin{pmatrix}3\\4\\-2\end{pmatrix}$ are perpendicular.

10 Without finding the angle, decide whether the angle between $\begin{pmatrix}1\\-3\\4\end{pmatrix}$ and $\begin{pmatrix}2\\5\\3\end{pmatrix}$ is acute, a right angle or obtuse.

11 $|\mathbf{a}|=4$, $|\mathbf{b}|=5$ and the angle between $\mathbf{a}$ and $\mathbf{b}$ is $60°$. Find $\mathbf{a}\cdot\mathbf{b}$ and $\mathbf{a}\cdot\mathbf{a}$.

12 Jo says that two vectors with $|\mathbf{a}|=2$ and $|\mathbf{b}|=5$ have $\mathbf{a}\cdot\mathbf{b}=12$. Explain why Jo must be wrong.

D · Algebra and "show that"

Set the scalar product equal to 0.

13 Find k so that $\begin{pmatrix}k\\3\\-2\end{pmatrix}$ and $\begin{pmatrix}4\\k\\6\end{pmatrix}$ are perpendicular.

14 Find the possible values of p for which $\begin{pmatrix}p\\4\\1\end{pmatrix}$ and $\begin{pmatrix}p\\p\\-12\end{pmatrix}$ are perpendicular.

15 Find a vector of the form $\begin{pmatrix}1\\p\\q\end{pmatrix}$ that is perpendicular to both $\begin{pmatrix}1\\1\\1\end{pmatrix}$ and $\begin{pmatrix}1\\2\\3\end{pmatrix}$.

16 The points $A(2,1,-1)$, $B(4,3,0)$ and $C(5,1,2)$ form a triangle. Show that the triangle has a right angle at B.

E · Exam-style question

Show your method: marks are for working.

17 The points A, B and C have coordinates $(2,0,1)$, $(4,3,-5)$ and $(5,-2,k)$ respectively, where k is a constant.

(a) Find $\overrightarrow{AB}\cdot\overrightarrow{AC}$ in terms of k. [2 marks]

(b) Angle BAC is a right angle. Find the value of k. [2 marks]

(c) Using your value of k, find angle ABC, giving your answer to the nearest $0.1°$. [4 marks]