Vector Equation of a Line

Specification: AQA 7366/7367: F1 · Edexcel 9FM0: Core Pure 6.1

INDEPENDENT

You do

A · Writing equations r = a + λd. Through two points: d = b − a.

1 Find a vector equation of the line through $(3, -2, 1)$ in the direction $\begin{pmatrix} 4 \\ 1 \\ -5 \end{pmatrix}$.

2 Find a vector equation of the line through $(0, 6, -1)$ in the direction $2\mathbf{i} - 3\mathbf{k}$.

3 Find a vector equation of the line through A$(1, -3, 2)$ and B$(4, 0, -1)$.

4 Find a vector equation of the line through P$(-2, 5, 0)$ and Q$(1, 5, 4)$.

B · Points on lines Find λ from one component. Check the other two.

5 Find the coordinates of the point on the line $\mathbf{r} = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} + \lambda\begin{pmatrix} 1 \\ 4 \\ -2 \end{pmatrix}$ where $\lambda = -3$.

6 Show that the point $(6, 7, -5)$ lies on the line $\mathbf{r} = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} + \lambda\begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}$.

7 Determine whether the point $(-1, 5, 8)$ lies on the line $\mathbf{r} = \begin{pmatrix} 5 \\ -1 \\ 2 \end{pmatrix} + \mu\begin{pmatrix} -3 \\ 3 \\ 2 \end{pmatrix}$.

8 The point $(p, 1, q)$ lies on the line $\mathbf{r} = \begin{pmatrix} 3 \\ -5 \\ 2 \end{pmatrix} + t\begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix}$. Find p and q.

C · Reasoning No calculator. Explain in words as well as numbers.

9 Show that $\mathbf{r} = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} + \lambda\begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}$ and $\mathbf{r} = \begin{pmatrix} 5 \\ 0 \\ 5 \end{pmatrix} + \mu\begin{pmatrix} -4 \\ 2 \\ -6 \end{pmatrix}$ are the same line.

10 The points A$(1, 1, 2)$ and B$(3, 5, -4)$ are given. Show that the line AB is parallel to the line $\mathbf{r} = \begin{pmatrix} 0 \\ 2 \\ 7 \end{pmatrix} + \lambda\begin{pmatrix} -1 \\ -2 \\ 3 \end{pmatrix}$.

11 Show that the line $\mathbf{r} = \begin{pmatrix} 3 \\ 0 \\ -1 \end{pmatrix} + \lambda\begin{pmatrix} 2 \\ -1 \\ 4 \end{pmatrix}$ is perpendicular to the vector $\begin{pmatrix} 1 \\ 6 \\ 1 \end{pmatrix}$.

12 The points A$(1, 2, 3)$, B$(3, -1, 5)$ and C$(7, k, 9)$ lie on a straight line. Find k.

D · Algebra and "show that" Parallel lines share a direction. A distance is a magnitude.

13 Find a vector equation of the line through $(4, -3, 1)$ that is parallel to the line $\mathbf{r} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + \mu\begin{pmatrix} 2 \\ -5 \\ 3 \end{pmatrix}$.

14 Find the coordinates of the point where the line $\mathbf{r} = \begin{pmatrix} 5 \\ -3 \\ 4 \end{pmatrix} + \lambda\begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}$ crosses the plane $z = 0$.

15 The line l passes through A$(2, -1, 5)$ and B$(6, 1, -1)$. The point C lies on l and B is the midpoint of AC. Find the coordinates of C.

16 The line l has equation $\mathbf{r} = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} + \lambda\begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix}$ and passes through A$(1, 2, -1)$. Find the coordinates of the two points on l that are a distance 6 from A.

E · Exam-style question Show your method: marks are for working.

17 The points A and B have coordinates $(3, -1, 2)$ and $(5, 3, -4)$ respectively.
(a) Find a vector equation of the line AB. [2 marks]
(b) Show that the point C$(0, -7, 11)$ lies on the line AB. [2 marks]
(c) The line l passes through D$(1, 4, 0)$ and is parallel to AB. Write down a vector equation of l. [1 mark]
(d) Show that l and the line AB are not the same line. [2 marks]