Cartesian Form of a Line

Specification: AQA 7366/7367: F1 · Edexcel 9FM0: Core Pure 6.1

INDEPENDENT **You do**

A · Vector → Cartesian Make λ the subject of each component. A direction component of 0 gives $x = a_1$.

1 Find a Cartesian equation of the line $\mathbf{r} = \begin{pmatrix} 4 \\ -1 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 2 \\ 5 \end{pmatrix}$.

2 Find a Cartesian equation of the line $\mathbf{r} = \begin{pmatrix} 0 \\ 3 \\ -5 \end{pmatrix} + \lambda \begin{pmatrix} -2 \\ 1 \\ 4 \end{pmatrix}$.

3 Find a Cartesian equation of the line $\mathbf{r} = (6\mathbf{i} - 2\mathbf{k}) + \lambda(5\mathbf{i} + 4\mathbf{j} - 3\mathbf{k})$.

4 Find a Cartesian equation of the line $\mathbf{r} = \begin{pmatrix} 1 \\ -4 \\ 3 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 0 \\ -5 \end{pmatrix}$.

B · Cartesian → vector Rewrite each fraction as $(x - a)/d$ first.

5 Find a vector equation of the line $\dfrac{x-5}{2} = \dfrac{y+4}{-3} = \dfrac{z-1}{6}$.

6 Find a vector equation of the line $\dfrac{x+1}{3} = \dfrac{4-y}{2} = \dfrac{z}{-5}$.

7 Find a vector equation of the line $\dfrac{3x-2}{6} = \dfrac{y+5}{4} = \dfrac{1-z}{3}$.

8 Find a vector equation of the line $x = -3, \dfrac{y-2}{7} = \dfrac{z+6}{2}$.

C · Using the Cartesian form Substitute into every fraction: all must give the same value.

9 Determine whether the point $(7, -1, -8)$ lies on the line $\dfrac{x-1}{3} = \dfrac{y+5}{2} = \dfrac{z+2}{-3}$.

10 The point $(p, q, 5)$ lies on the line $\dfrac{x+2}{4} = \dfrac{y-3}{-1} = \dfrac{z-1}{2}$. Find p and q.

11 Find the coordinates of the point where the line $\dfrac{x-3}{2} = \dfrac{y+1}{-4} = \dfrac{z-6}{3}$ meets the plane $z = 0$.

12 Find a Cartesian equation of the line through $A(2, 5, -1)$ and $B(4, 1, 5)$.

D · Reasoning No calculator. Parallel lines: directions are multiples. Explain in words as well as numbers.

13 Show that the line $\dfrac{x-1}{2} = \dfrac{y+4}{-1} = \dfrac{z}{3}$ is perpendicular to the vector $4\mathbf{i} + 5\mathbf{j} - \mathbf{k}$ (compare Do Now Q4).

14 Show that l_1: $\dfrac{x-2}{3} = \dfrac{y+1}{-1} = \dfrac{z-4}{2}$ and l_2: $\dfrac{8-x}{6} = \dfrac{y+3}{2} = \dfrac{z-8}{-4}$ are the same line.

15 Find a Cartesian equation of the line through $(-1, 2, 0)$ that is parallel to the line $\dfrac{x+4}{3} = \dfrac{1-y}{2}, z = 5$.

16 The lines $\dfrac{x-1}{k} = \dfrac{y+2}{3} = \dfrac{z-5}{-6}$ and $\dfrac{x+4}{2} = \dfrac{y}{-1} = \dfrac{z-3}{2}$ are parallel. Find k.

E · Exam-style question Show your method: marks are for working.

17 The line l has Cartesian equation $\dfrac{x-4}{2} = \dfrac{1-y}{3} = z + 2$.

(a) Find a vector equation of l. [2 marks]

(b) Find the coordinates of the point where l meets the plane $z = 0$. [2 marks]

(c) Show that the point $A(0, 7, -4)$ lies on l. [2 marks]

(d) Show that l is perpendicular to the vector $3\mathbf{i} + \mathbf{j} - 3\mathbf{k}$. [2 marks]