# Angle Between Two Lines

Specification: AQA 7366/7367: F1, F4 · Edexcel 9FM0: Core Pure 6.3, 6.4

## HOMEWORK Consolidation

**1** Find the acute angle between the lines $l_1$: $\mathbf{r} = \begin{pmatrix}1\\3\\0\end{pmatrix} + \lambda\begin{pmatrix}2\\-1\\3\end{pmatrix}$ and $l_2$: $\mathbf{r} = \begin{pmatrix}2\\0\\-1\end{pmatrix} + \mu\begin{pmatrix}1\\4\\-1\end{pmatrix}$.

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**2** Find the acute angle between the lines $\mathbf{r} = (2\mathbf{i} + \mathbf{k}) + \lambda(\mathbf{i} + 2\mathbf{j} - \mathbf{k})$ and $\mathbf{r} = (\mathbf{j} - \mathbf{k}) + \mu(3\mathbf{i} - \mathbf{j} + 2\mathbf{k})$.

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**3** Find the acute angle between the lines $l_1$: $\frac{x-3}{2} = \frac{y+1}{6} = \frac{z-2}{-3}$ and $l_2$: $\frac{x+2}{3} = 4 - y = z - 1$.

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**4** Find the acute angle between the lines $l_1$: $\mathbf{r} = \begin{pmatrix}1\\1\\1\end{pmatrix} + \lambda\begin{pmatrix}4\\-1\\1\end{pmatrix}$ and $l_2$: $\frac{3-x}{2} = \frac{2y+1}{4} = z$.

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**5** The line $l_1$ passes through A$(0, 2, -1)$ and B$(2, 5, 5)$. The line $l_2$ has Cartesian equation $x = 4$, $\frac{y-1}{3} = \frac{z+2}{4}$. Find the acute angle between $l_1$ and $l_2$.

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**6** The line $l$ is $\mathbf{r} = \begin{pmatrix}1\\-2\\0\end{pmatrix} + \lambda\begin{pmatrix}1\\-4\\8\end{pmatrix}$.

(a) Find the acute angle between $l$ and the $x$-axis.

(b) Find the acute angle between $l$ and the $z$-axis.

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**7** Show that the lines $\frac{x-2}{3} = \frac{y+1}{-2} = \frac{z-4}{5}$ and $\mathbf{r} = \begin{pmatrix}0\\3\\1\end{pmatrix} + \mu\begin{pmatrix}1\\4\\1\end{pmatrix}$ are perpendicular.

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**8** The lines $\mathbf{r} = \begin{pmatrix}1\\0\\2\end{pmatrix} + \lambda\begin{pmatrix}2\\a\\a-1\end{pmatrix}$ and $\mathbf{r} = \begin{pmatrix}3\\-1\\0\end{pmatrix} + \mu\begin{pmatrix}3\\-1\\2\end{pmatrix}$ are perpendicular. Find $a$.

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**9** The line $l_1$: $\frac{x+1}{p} = \frac{y-2}{3} = \frac{z-1}{-2}$ is perpendicular to $\mathbf{r} = \begin{pmatrix}2\\0\\-3\end{pmatrix} + \mu\begin{pmatrix}4\\-2\\p\end{pmatrix}$. Find $p$.

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**10** The lines $l_1$: $\mathbf{r} = \begin{pmatrix}2\\1\\0\end{pmatrix} + \lambda\begin{pmatrix}1\\-2\\2\end{pmatrix}$ and $l_2$: $\mathbf{r} = \begin{pmatrix}0\\3\\1\end{pmatrix} + \mu\begin{pmatrix}3\\-6\\2\end{pmatrix}$ are skew. Find the acute angle between them.

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**11** *AQA-style*. The points A and B have coordinates $(1, 3, -2)$ and $(4, -1, -2)$ respectively. The line $l$ has Cartesian equation $\frac{x-2}{6} = \frac{y+1}{2} = \frac{3-z}{3}$.

(a) Find a vector equation of the line AB. [2 marks]

(b) Find the acute angle between AB and $l$, giving your answer to the nearest $0.1°$. [3 marks]

(c) Show that AB is perpendicular to the $z$-axis. [2 marks]

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**12** *Edexcel-style*. The lines $l_1$ and $l_2$ have equations

$l_1$: $\mathbf{r} = (2\mathbf{i} - \mathbf{j} + \mathbf{k}) + \lambda(\mathbf{i} + 2\mathbf{j} - 2\mathbf{k})$

$l_2$: $\mathbf{r} = (\mathbf{i} + 3\mathbf{k}) + \mu(4\mathbf{i} + a\mathbf{j} + \mathbf{k})$

where $a$ is a constant.

(a) Given that $l_1$ and $l_2$ are perpendicular, find the value of $a$. (2)

(b) The line $l_3$ has equation $\frac{x-1}{2} = \frac{y+2}{-3} = \frac{z}{6}$. Find the acute angle between $l_1$ and $l_3$, giving your answer to one decimal place. (4)

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