Angle Between Two Lines

Specification: AQA 7366/7367: F1, F4 · Edexcel 9FM0: Core Pure 6.3, 6.4

INDEPENDENT **You do**

A · Lines in vector form Use the direction vectors only. A negative scalar product: take the modulus.

1 Find the acute angle between the lines l_1: $\mathbf{r} = \begin{pmatrix}1\\0\\3\end{pmatrix} + \lambda\begin{pmatrix}2\\2\\-1\end{pmatrix}$ and l_2: $\mathbf{r} = \begin{pmatrix}0\\-2\\1\end{pmatrix} + \mu\begin{pmatrix}6\\-3\\2\end{pmatrix}$.

2 Find the acute angle between the lines l_1: $\mathbf{r} = \begin{pmatrix}2\\5\\-1\end{pmatrix} + \lambda\begin{pmatrix}4\\-4\\7\end{pmatrix}$ and l_2: $\mathbf{r} = \begin{pmatrix}1\\1\\1\end{pmatrix} + \mu\begin{pmatrix}2\\-1\\-2\end{pmatrix}$.

3 Find the acute angle between the lines l_1: $\mathbf{r} = (\mathbf{i} - \mathbf{j}) + \lambda(3\mathbf{i} - 4\mathbf{k})$ and l_2: $\mathbf{r} = (2\mathbf{j} + \mathbf{k}) + \mu(\mathbf{i} - 2\mathbf{j} + 2\mathbf{k})$.

4 Find the acute angle between the lines l_1: $\mathbf{r} = \begin{pmatrix}0\\2\\-3\end{pmatrix} + \lambda\begin{pmatrix}1\\3\\-2\end{pmatrix}$ and l_2: $\mathbf{r} = \begin{pmatrix}4\\0\\1\end{pmatrix} + \mu\begin{pmatrix}2\\-1\\4\end{pmatrix}$.

B · Cartesian and mixed forms Rearrange awkward fractions first, then read the directions (Lesson 3).

5 Find the acute angle between the lines l_1: $\dfrac{x-2}{3} = \dfrac{y+1}{-2} = \dfrac{z}{6}$ and l_2: $x + 4 = \dfrac{y-5}{4} = \dfrac{z-1}{8}$.

6 Find the acute angle between the lines l_1: $\dfrac{1-x}{2} = \dfrac{y+3}{2} = \dfrac{2z-1}{2}$ and l_2: $\mathbf{r} = \begin{pmatrix}0\\1\\1\end{pmatrix} + \mu\begin{pmatrix}4\\0\\-3\end{pmatrix}$.

7 Find the acute angle between the lines l_1: $x = 3$, $\dfrac{y-1}{4} = \dfrac{z+2}{-3}$ and l_2: $x + 1 = \dfrac{y}{2} = \dfrac{z-2}{2}$.

8 The line l_1 passes through A$(1, 0, -2)$ and B$(3, -1, 0)$. The line l_2 is $x - 4 = \dfrac{y}{4} = \dfrac{z+3}{-8}$. Find the acute angle between l_1 and l_2.

C · Perpendicular lines Perpendicular ⇔ $\mathbf{d}_1 \cdot \mathbf{d}_2 = 0$: set it equal to 0 and solve.

9 Show that the lines l_1: $\mathbf{r} = \begin{pmatrix}1\\2\\3\end{pmatrix} + \lambda\begin{pmatrix}2\\-1\\3\end{pmatrix}$ and l_2: $\dfrac{x-1}{4} = \dfrac{y+2}{5} = \dfrac{z-1}{-1}$ are perpendicular.

10 The lines $\mathbf{r} = \begin{pmatrix}2\\0\\1\end{pmatrix} + \lambda\begin{pmatrix}a\\2\\-1\end{pmatrix}$ and $\mathbf{r} = \begin{pmatrix}-1\\3\\0\end{pmatrix} + \mu\begin{pmatrix}3\\-4\\a\end{pmatrix}$ are perpendicular. Find a.

11 The line l_1: $\dfrac{x-1}{p} = \dfrac{y+3}{2} = \dfrac{z}{p+1}$ is perpendicular to l_2: $\mathbf{r} = \begin{pmatrix}0\\1\\4\end{pmatrix} + \mu\begin{pmatrix}2\\-5\\1\end{pmatrix}$. Find p.

12 The points A$(1, k, 0)$ and B$(3, 1, 2)$ lie on the line l_1, which is perpendicular to l_2: $\mathbf{r} = \begin{pmatrix}5\\0\\1\end{pmatrix} + \mu\begin{pmatrix}3\\2\\-1\end{pmatrix}$. Find k.

D · Axes, skew lines and reasoning Explain in words as well as numbers.

13 The line l_1 has direction $\mathbf{i} + 4\mathbf{j} + \mathbf{k}$. Beth writes the direction of l_2 as $2\mathbf{i} - \mathbf{j} + 3\mathbf{k}$; Amir writes it as $-4\mathbf{i} + 2\mathbf{j} - 6\mathbf{k}$.
(a) Use Beth's vector to find the acute angle between l_1 and l_2.
(b) Show that Amir's vector gives the same angle.

14 The lines l_1: $\mathbf{r} = \begin{pmatrix}1\\0\\2\end{pmatrix} + \lambda\begin{pmatrix}6\\2\\-3\end{pmatrix}$ and l_2: $\mathbf{r} = \begin{pmatrix}0\\1\\-1\end{pmatrix} + \mu\begin{pmatrix}0\\4\\3\end{pmatrix}$ are skew. Hana says skew lines have no angle between them. Explain why she is wrong, and find the acute angle.

15 Find the acute angle between the line $\dfrac{x+2}{2} = \dfrac{1-y}{3} = \dfrac{z-4}{6}$ and the y-axis.

16 The line l has direction $a\mathbf{i} + 3\mathbf{j} - 4\mathbf{k}$ and is perpendicular to the x-axis.
(a) Find a.
(b) Find the acute angle between l and the y-axis.

E · Exam-style question Show your method: marks are for working.

17 The lines l_1 and l_2 have equations l_1: $\mathbf{r} = \begin{pmatrix}2\\-1\\3\end{pmatrix} + \lambda\begin{pmatrix}1\\2\\-2\end{pmatrix}$ and l_2: $\dfrac{x-5}{4} = \dfrac{y+1}{-1} = \dfrac{z-2}{8}$.
(a) Find the acute angle between l_1 and l_2, giving your answer to the nearest 0.1°. [3 marks]
(b) The line l_3 has direction $4\mathbf{i} + p\mathbf{j} + \mathbf{k}$ and is perpendicular to l_1. Find p. [2 marks]
(c) A student uses $2\mathbf{i} - \mathbf{j} + 3\mathbf{k}$ to find the angle in part (a). Explain why this is wrong. [1 mark]
(d) Find the acute angle between l_2 and the z-axis. [2 marks]