Intersecting, Parallel or Skew Lines

Specification: AQA 7366/7367: F1 · Edexcel 9FM0: Core Pure 6.5

INDEPENDENT **You do**

A · Lines that meet Write the three equations, solve two, check the third, then substitute.

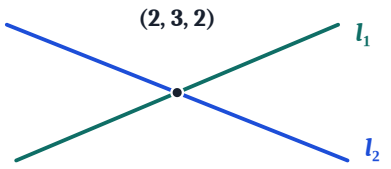
1 l_1: $\mathbf{r} = \begin{pmatrix}1\\2\\0\end{pmatrix} + \lambda\begin{pmatrix}1\\1\\2\end{pmatrix}$ and l_2: $\mathbf{r} = \begin{pmatrix}5\\4\\3\end{pmatrix} + \mu\begin{pmatrix}3\\1\\1\end{pmatrix}$.
Show that they intersect, and find the point.

Directions $\begin{pmatrix}1\\1\\2\end{pmatrix}$ and $\begin{pmatrix}3\\1\\1\end{pmatrix}$ are not multiples, so the lines are not parallel. — compare the directions

x: $1 + \lambda = 5 + 3\mu \Rightarrow \lambda - 3\mu = 4$ (1)
y: $2 + \lambda = 4 + \mu \Rightarrow \lambda - \mu = 2$ (2) — components equal: λ, μ
z: $2\lambda = 3 + \mu$ (3)
(2) − (1): $2\mu = -2 \Rightarrow \mu = -1$ — subtract: λ cancels
(1): $\lambda - 3(-1) = 4 \Rightarrow \lambda = 1$ — substitute: find λ
(3): $2(1) = 2, 3 - 1 = 2$ ✓ — check the third: ✓

$\mathbf{r} = \begin{pmatrix}1\\2\\0\end{pmatrix} + \begin{pmatrix}1\\1\\2\end{pmatrix} = \begin{pmatrix}2\\3\\2\end{pmatrix}$: they meet at $(2, 3, 2)$

substitute λ: the point



picture: they meet at the point

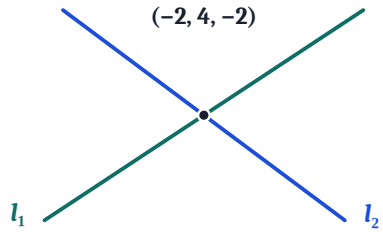
2 l_1: $\mathbf{r} = \begin{pmatrix}0\\3\\-1\end{pmatrix} + \lambda\begin{pmatrix}2\\-1\\1\end{pmatrix}$ and l_2: $\mathbf{r} = \begin{pmatrix}-4\\0\\4\end{pmatrix} + \mu\begin{pmatrix}1\\2\\-3\end{pmatrix}$.
Show that they intersect, and find the point.

Directions $\begin{pmatrix}2\\-1\\1\end{pmatrix}$ and $\begin{pmatrix}1\\2\\-3\end{pmatrix}$ are not multiples, so the lines are not parallel. — compare the directions

x: $2\lambda = -4 + \mu \Rightarrow 2\lambda - \mu = -4$ (1)
y: $3 - \lambda = 2\mu \Rightarrow \lambda + 2\mu = 3$ (2) — components equal: λ, μ
z: $-1 + \lambda = 4 - 3\mu$ (3)
$2 \times (1) + (2)$: $5\lambda = -5 \Rightarrow \lambda = -1$ — add: μ cancels
(1): $2(-1) - \mu = -4 \Rightarrow \mu = 2$ — substitute: find μ
(3): $-1 - 1 = -2, 4 - 3(2) = -2$ ✓ — check the third: ✓

$\mathbf{r} = \begin{pmatrix}0\\3\\-1\end{pmatrix} - \begin{pmatrix}2\\-1\\1\end{pmatrix} = \begin{pmatrix}-2\\4\\-2\end{pmatrix}$: they meet at $(-2, 4, -2)$

substitute λ: the point



picture: they meet at the point

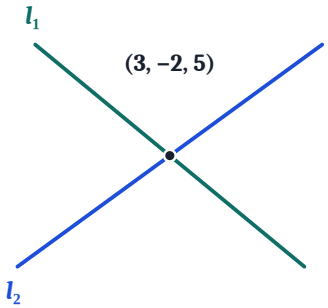
3 l_1: $\mathbf{r} = (\mathbf{i} + \mathbf{k}) + \lambda(\mathbf{i} - \mathbf{j} + 2\mathbf{k})$ and
l_2: $\mathbf{r} = (3\mathbf{i} - 5\mathbf{j} + 2\mathbf{k}) + \mu(\mathbf{j} + \mathbf{k})$.
Show that they intersect, and find the point.

Directions $\begin{pmatrix}1\\-1\\2\end{pmatrix}$ and $\begin{pmatrix}0\\1\\1\end{pmatrix}$ are not multiples, so the lines are not parallel. — compare the directions

x: $1 + \lambda = 3 \Rightarrow \lambda = 2$ (1)
y: $-\lambda = -5 + \mu \Rightarrow \lambda + \mu = 5$ (2) — components equal: λ, μ
z: $1 + 2\lambda = 2 + \mu$ (3)
(2): $2 + \mu = 5 \Rightarrow \mu = 3$ — substitute: find μ
(3): $1 + 2(2) = 5, 2 + 3 = 5$ ✓ — check the third: ✓

$\mathbf{r} = \begin{pmatrix}1\\0\\1\end{pmatrix} + 2\begin{pmatrix}1\\-1\\2\end{pmatrix} = \begin{pmatrix}3\\-2\\5\end{pmatrix}$: they meet at $(3, -2, 5)$

substitute λ: the point



picture: they meet at the point

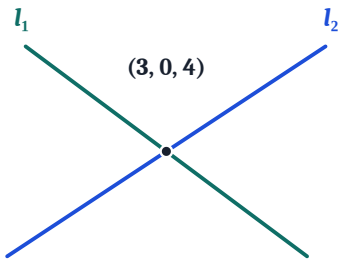
4 l_1: $\mathbf{r} = \begin{pmatrix}4\\-2\\1\end{pmatrix} + \lambda\begin{pmatrix}-1\\2\\3\end{pmatrix}$ and l_2: $\mathbf{r} = \begin{pmatrix}7\\2\\2\end{pmatrix} + \mu\begin{pmatrix}2\\1\\-1\end{pmatrix}$.
Show that they intersect, and find the point.

Directions $\begin{pmatrix}-1\\2\\3\end{pmatrix}$ and $\begin{pmatrix}2\\1\\-1\end{pmatrix}$ are not multiples, so the lines are not parallel. — compare the directions

x: $4 - \lambda = 7 + 2\mu \Rightarrow \lambda + 2\mu = -3$ (1)
y: $-2 + 2\lambda = 2 + \mu \Rightarrow 2\lambda - \mu = 4$ (2) — components equal: λ, μ
z: $1 + 3\lambda = 2 - \mu$ (3)
(1) + 2 × (2): $5\lambda = 5 \Rightarrow \lambda = 1$ — add: μ cancels
(2): $2(1) - \mu = 4 \Rightarrow \mu = -2$ — substitute: find μ
(3): $1 + 3(1) = 4, 2 + 2 = 4$ ✓ — check the third: ✓

$\mathbf{r} = \begin{pmatrix}4\\-2\\1\end{pmatrix} + \begin{pmatrix}-1\\2\\3\end{pmatrix} = \begin{pmatrix}3\\0\\4\end{pmatrix}$: they meet at $(3, 0, 4)$

substitute λ: the point



picture: they meet at the point

B · Meet or skew? The third equation decides.

5 l_1: $\mathbf{r} = \begin{pmatrix}1\\1\\1\end{pmatrix} + \lambda\begin{pmatrix}1\\2\\-1\end{pmatrix}$ and l_2: $\mathbf{r} = \begin{pmatrix}0\\2\\1\end{pmatrix} + \mu\begin{pmatrix}2\\1\\1\end{pmatrix}$.

Intersect or skew? If they meet, find the point.

Directions $\begin{pmatrix}1\\2\\-1\end{pmatrix}$ and $\begin{pmatrix}2\\1\\1\end{pmatrix}$ are not multiples, so the lines are not parallel.

compare the directions

x: $1 + \lambda = 2\mu \Rightarrow \lambda - 2\mu = -1$ (1)

y: $1 + 2\lambda = 2 + \mu \Rightarrow 2\lambda - \mu = 1$ (2) — components equal: λ, μ

z: $1 - \lambda = 1 + \mu$ (3)

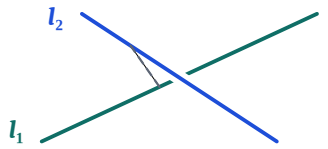
$2 \times (2) - (1)$: $3\lambda = 3 \Rightarrow \lambda = 1$ — subtract: μ cancels

(2): $2(1) - \mu = 1 \Rightarrow \mu = 1$ — substitute: find μ

(3): $1 - 1 = 0, 1 + 1 = 2, 0 \neq 2$ — check the third: ≠

(3) is not true, so there is no common point. Not parallel and no common point: the lines are **skew**.

≠ and not parallel: skew



picture: skew, one passes over the other

6 l_1: $\mathbf{r} = \begin{pmatrix}2\\0\\-3\end{pmatrix} + \lambda\begin{pmatrix}1\\-2\\4\end{pmatrix}$ and l_2: $\mathbf{r} = \begin{pmatrix}-1\\0\\-3\end{pmatrix} + \mu\begin{pmatrix}1\\1\\-2\end{pmatrix}$.

Intersect or skew? If they meet, find the point.

Directions $\begin{pmatrix}1\\-2\\4\end{pmatrix}$ and $\begin{pmatrix}1\\1\\-2\end{pmatrix}$ are not multiples, so the lines are not parallel.

compare the directions

x: $2 + \lambda = -1 + \mu \Rightarrow \lambda - \mu = -3$ (1)

y: $-2\lambda = \mu \Rightarrow 2\lambda + \mu = 0$ (2) — components equal: λ, μ

z: $-3 + 4\lambda = -3 - 2\mu$ (3)

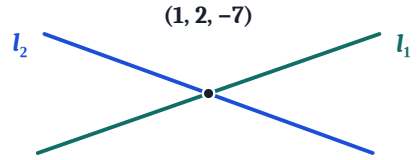
(1) + (2): $3\lambda = -3 \Rightarrow \lambda = -1$ — add: μ cancels

(1): $-1 - \mu = -3 \Rightarrow \mu = 2$ — substitute: find μ

(3): $-3 + 4(-1) = -7, -3 - 2(2) = -7$ ✓ — check the third: ✓

$\mathbf{r} = \begin{pmatrix}2\\0\\-3\end{pmatrix} - \begin{pmatrix}1\\-2\\4\end{pmatrix} = \begin{pmatrix}1\\2\\-7\end{pmatrix}$: they meet at $(1, 2, -7)$

substitute λ: the point



picture: they meet at the point

7 l_1: $\mathbf{r} = \begin{pmatrix}0\\1\\2\end{pmatrix} + \lambda\begin{pmatrix}3\\0\\1\end{pmatrix}$ and l_2: $\mathbf{r} = \begin{pmatrix}4\\0\\2\end{pmatrix} + \mu\begin{pmatrix}1\\-1\\2\end{pmatrix}$.

Intersect or skew? If they meet, find the point.

Directions $\begin{pmatrix}3\\0\\1\end{pmatrix}$ and $\begin{pmatrix}1\\-1\\2\end{pmatrix}$ are not multiples, so the lines are not parallel.

compare the directions

x: $3\lambda = 4 + \mu \Rightarrow 3\lambda - \mu = 4$ (1)

y: $1 = -\mu \Rightarrow \mu = -1$ (2) — components equal: λ, μ

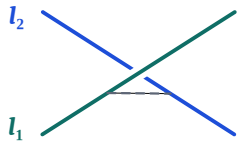
z: $2 + \lambda = 2 + 2\mu$ (3)

(1): $3\lambda + 1 = 4 \Rightarrow \lambda = 1$ — substitute: find λ

(3): $2 + 1 = 3, 2 + 2(-1) = 0, 3 \neq 0$ — check the third: ≠

(3) is not true, so there is no common point. Not parallel and no common point: the lines are **skew**.

≠ and not parallel: skew



picture: skew, one passes over the other

8 l_1: $\mathbf{r} = \begin{pmatrix}1\\1\\0\end{pmatrix} + \lambda\begin{pmatrix}1\\-1\\1\end{pmatrix}$ and l_2: $\dfrac{x-1}{2} = \dfrac{y+4}{3} = 3 - z$.

Intersect or skew? If they meet, find the point.

$3 - z = \dfrac{z-3}{-1}$; l_2: point $(1, -4, 3)$, direction $\begin{pmatrix}2\\3\\-1\end{pmatrix}$; not a multiple of $\begin{pmatrix}1\\-1\\1\end{pmatrix}$, so not parallel

rewrite; point and direction

x: $1 + \lambda = 1 + 2\mu \Rightarrow \lambda - 2\mu = 0$ (1)

y: $1 - \lambda = -4 + 3\mu \Rightarrow \lambda + 3\mu = 5$ (2) — components equal: λ, μ

z: $\lambda = 3 - \mu$ (3)

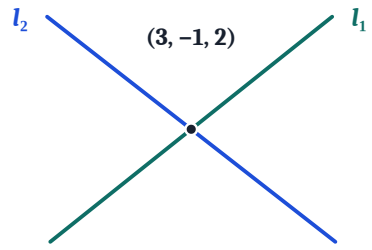
(2) − (1): $5\mu = 5 \Rightarrow \mu = 1$ — subtract: λ cancels

(1): $\lambda - 2(1) = 0 \Rightarrow \lambda = 2$ — substitute: find λ

(3): $2 = 2, 3 - 1 = 2$ ✓ — check the third: ✓

$\mathbf{r} = \begin{pmatrix}1\\1\\0\end{pmatrix} + 2\begin{pmatrix}1\\-1\\1\end{pmatrix} = \begin{pmatrix}3\\-1\\2\end{pmatrix}$: they meet at $(3, -1, 2)$

substitute λ: the point



picture: they meet at the point

C · Parallel, the same line, intersecting or skew Compare the directions first.

9 l_1: $\mathbf{r} = \begin{pmatrix}1\\2\\3\end{pmatrix} + \lambda\begin{pmatrix}2\\-1\\4\end{pmatrix}$ and l_2: $\mathbf{r} = \begin{pmatrix}3\\0\\5\end{pmatrix} + \mu\begin{pmatrix}-4\\2\\-8\end{pmatrix}$.

Parallel, the same line, intersecting or skew?

$\begin{pmatrix}-4\\2\\-8\end{pmatrix} = -2\begin{pmatrix}2\\-1\\4\end{pmatrix}$: the directions are multiples, so the lines are parallel. Is $(3, 0, 5)$ on l_1?

multiple ⇒ parallel

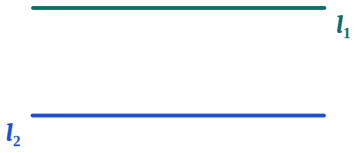
x: $1 + 2\lambda = 3 \Rightarrow \lambda = 1$ — x: find λ

y: $2 - \lambda = 0 \Rightarrow \lambda = 2$, not 1 — y: different λ

z: $3 + 4\lambda = 5 \Rightarrow \lambda = \frac{1}{2}$, not 1 — z: different λ

$(3, 0, 5)$ is not on l_1: parallel with no common point, so the lines are **parallel** and never meet.

parallel, no common point



picture: parallel, never meet

10 l_1: $\frac{x-1}{3} = \frac{y+2}{-1} = \frac{z-4}{2}$ and l_2: $\mathbf{r} = \begin{pmatrix}7\\-4\\8\end{pmatrix} + \mu\begin{pmatrix}6\\-2\\4\end{pmatrix}$.

Parallel, the same line, intersecting or skew?

l_1: point $(1, -2, 4)$, direction $\begin{pmatrix}3\\-1\\2\end{pmatrix}$ — Cartesian: point, direction

$\begin{pmatrix}6\\-2\\4\end{pmatrix} = 2\begin{pmatrix}3\\-1\\2\end{pmatrix}$: the directions are multiples, so the lines are parallel. Is $(7, -4, 8)$ on l_1?

multiple ⇒ parallel

x: $1 + 3\lambda = 7 \Rightarrow \lambda = 2$ — x: find λ

y: $-2 - \lambda = -4 \Rightarrow \lambda = 2$ ✓ — y: same λ

z: $4 + 2\lambda = 8 \Rightarrow \lambda = 2$ ✓ — z: same λ

$(7, -4, 8)$ is on l_1 too: parallel with a common point, so they are the **same line**.

parallel + common point: same



picture: the same line

11 l_1: $x - 2 = \frac{y-1}{2} = \frac{z+1}{-1}$ and l_2: $\frac{x-6}{3} = y - 4 = z - 1$.

Parallel, the same line, intersecting or skew?

l_1: point $(2, 1, -1)$, direction $\begin{pmatrix}1\\2\\-1\end{pmatrix}$ — Cartesian: point, direction

l_2: point $(6, 4, 1)$, direction $\begin{pmatrix}3\\1\\1\end{pmatrix}$; not a multiple of $\begin{pmatrix}1\\2\\-1\end{pmatrix}$, so not parallel

Cartesian: point, direction

x: $2 + \lambda = 6 + 3\mu \Rightarrow \lambda - 3\mu = 4$ (1)

y: $1 + 2\lambda = 4 + \mu \Rightarrow 2\lambda - \mu = 3$ (2) — components equal: λ, μ

z: $-1 - \lambda = 1 + \mu$ (3)

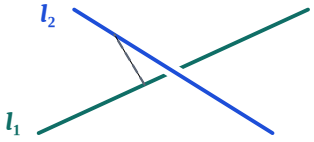
(2) − 2 × (1): $5\mu = -5 \Rightarrow \mu = -1$ — subtract: λ cancels

(1): $\lambda - 3(-1) = 4 \Rightarrow \lambda = 1$ — substitute: find λ

(3): $-1 - 1 = -2, 1 - 1 = 0, -2 \neq 0$ — check the third: ≠

(3) is not true, so there is no common point. Not parallel and no common point: the lines are **skew**.

≠ and not parallel: skew



picture: skew, one passes over the other

12 l_1: $\mathbf{r} = (3\mathbf{i} - \mathbf{j} + 2\mathbf{k}) + \lambda(\mathbf{i} - 2\mathbf{k})$ and
l_2: $\mathbf{r} = (3\mathbf{i} - 3\mathbf{j} + 4\mathbf{k}) + \mu(-\mathbf{i} + \mathbf{j} + \mathbf{k})$.
Parallel, the same line, intersecting or skew?

Directions $\begin{pmatrix}1\\0\\-2\end{pmatrix}$ and $\begin{pmatrix}-1\\1\\1\end{pmatrix}$ are not multiples, so the lines are not parallel.

compare the directions

x: $3 + \lambda = 3 - \mu \Rightarrow \lambda + \mu = 0$ (1)

y: $-1 = -3 + \mu \Rightarrow \mu = 2$ (2) — components equal: λ, μ

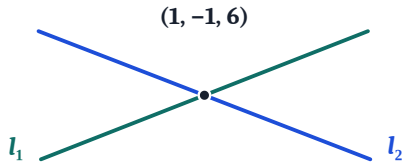
z: $2 - 2\lambda = 4 + \mu$ (3)

(1): $\lambda + 2 = 0 \Rightarrow \lambda = -2$ — substitute: find λ

(3): $2 - 2(-2) = 6, 4 + 2 = 6$ ✓ — check the third: ✓

$\mathbf{r} = \begin{pmatrix}3\\-1\\2\end{pmatrix} - 2\begin{pmatrix}1\\0\\-2\end{pmatrix} = \begin{pmatrix}1\\-1\\6\end{pmatrix}$: they meet at $(1, -1, 6)$

substitute λ: the point



picture: they meet at the point

D · Unknowns, angles and axes Solve the two equations without the unknown first.

13 l_1: $\mathbf{r} = \begin{pmatrix}1\\k\\2\end{pmatrix} + \lambda\begin{pmatrix}2\\1\\-1\end{pmatrix}$ and l_2: $\mathbf{r} = \begin{pmatrix}1\\2\\-1\end{pmatrix} + \mu\begin{pmatrix}1\\-1\\1\end{pmatrix}$ intersect. Find k and the point.

Directions $\begin{pmatrix}2\\1\\-1\end{pmatrix}$ and $\begin{pmatrix}1\\-1\\1\end{pmatrix}$ are not multiples, so the lines are not parallel.

compare the directions

x: $1 + 2\lambda = 1 + \mu \Rightarrow 2\lambda - \mu = 0$ (1)

y: $k + \lambda = 2 - \mu$ (2) — components equal: λ, μ

z: $2 - \lambda = -1 + \mu \Rightarrow \lambda + \mu = 3$ (3)

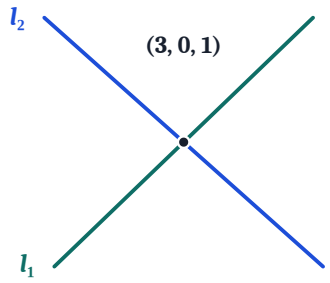
(1) + (3): $3\lambda = 3 \Rightarrow \lambda = 1$ — add: μ cancels

(1): $2(1) - \mu = 0 \Rightarrow \mu = 2$ — substitute: find μ

(2): $k + 1 = 2 - 2 \Rightarrow k = -1$ — third equation gives k

$\mathbf{r} = \begin{pmatrix}1\\-1\\2\end{pmatrix} + \begin{pmatrix}2\\1\\-1\end{pmatrix} = \begin{pmatrix}3\\0\\1\end{pmatrix}$: they meet at $(3, 0, 1)$

substitute λ: the point



picture: they meet at the point

14 l_1: $\mathbf{r} = \begin{pmatrix}0\\1\\-1\end{pmatrix} + \lambda\begin{pmatrix}1\\-2\\2\end{pmatrix}$ and l_2: $\mathbf{r} = \begin{pmatrix}3\\0\\-1\end{pmatrix} + \mu\begin{pmatrix}2\\1\\-2\end{pmatrix}$.

(a) Show that they intersect.

(b) Find the acute angle between them.

(a) x: $\lambda = 3 + 2\mu \Rightarrow \lambda - 2\mu = 3$ (1)

y: $1 - 2\lambda = \mu \Rightarrow 2\lambda + \mu = 1$ (2) — components equal: λ, μ

z: $-1 + 2\lambda = -1 - 2\mu$ (3)

(1) + 2 × (2): $5\lambda = 5 \Rightarrow \lambda = 1$ — add: μ cancels

(2): $2(1) + \mu = 1 \Rightarrow \mu = -1$ — substitute: find μ

(3): $-1 + 2(1) = 1$, $-1 - 2(-1) = 1$ ✓ — check the third: ✓

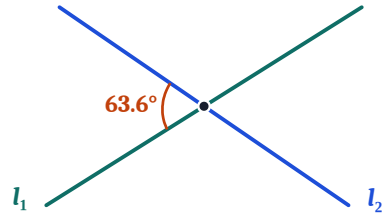
All three equations hold ($\lambda = 1$, $\mu = -1$), so the lines meet.

all three hold: they meet

(b) $\mathbf{d}_1 \cdot \mathbf{d}_2 = 2 - 2 - 4 = -4$, $|\mathbf{d}_1| = 3$, $|\mathbf{d}_2| = 3$

directions: $d_1 \cdot d_2$, $|d_1|$, $|d_2|$

$\cos\theta = \dfrac{|-4|}{3 \times 3}$, $\theta = 63.6°$ — modulus; inverse cos



picture: they meet; the angle

15 Show that the line $\mathbf{r} = \begin{pmatrix}2\\-4\\1\end{pmatrix} + \lambda\begin{pmatrix}1\\-2\\3\end{pmatrix}$ meets the z-axis, and find where.

The z-axis is $\mathbf{r} = \mu\mathbf{k}$: every point has $x = 0, y = 0$.

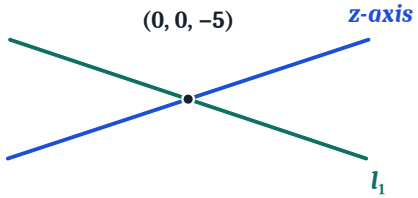
z-axis: x = 0, y = 0

x: $2 + \lambda = 0 \Rightarrow \lambda = -2$ — x: find λ

y: $-4 - 2\lambda = -4 + 4 = 0$ ✓ — y: same λ ✓

$z = 1 + 3(-2) = -5$: the line meets the z-axis at $(0, 0, -5)$

z: the point



picture: the line meets the z-axis

16 l_1: $\dfrac{x-3}{2} = y - k = \dfrac{z+1}{-2}$ and l_2: $\mathbf{r} = \begin{pmatrix}3\\3\\3\end{pmatrix} + \mu\begin{pmatrix}1\\1\\1\end{pmatrix}$ intersect. Find k and the point.

l_1: point $(3, k, -1)$, direction $\begin{pmatrix}2\\1\\-2\end{pmatrix}$ — Cartesian: point, direction

x: $3 + 2\lambda = 3 + \mu \Rightarrow 2\lambda - \mu = 0$ (1)

y: $k + \lambda = 3 + \mu$ (2) — components equal: λ, μ

z: $-1 - 2\lambda = 3 + \mu \Rightarrow 2\lambda + \mu = -4$ (3)

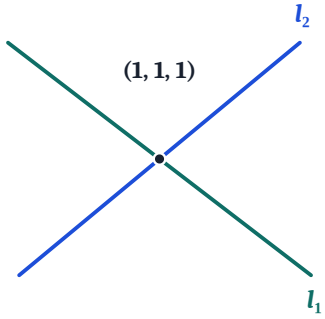
(1) + (3): $4\lambda = -4 \Rightarrow \lambda = -1$ — add: μ cancels

(1): $2(-1) - \mu = 0 \Rightarrow \mu = -2$ — substitute: find μ

(2): $k - 1 = 3 - 2 \Rightarrow k = 2$ — third equation gives k

$\mathbf{r} = \begin{pmatrix}3\\2\\-1\end{pmatrix} - \begin{pmatrix}2\\1\\-2\end{pmatrix} = \begin{pmatrix}1\\1\\1\end{pmatrix}$: they meet at $(1, 1, 1)$

substitute λ: the point



picture: they meet at the point

E · Exam-style question Show your method: marks are for working.

17 The lines l_1 and l_2 have equations

l_1: $\mathbf{r} = \begin{pmatrix} 1 \\ -3 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}$ and l_2: $\mathbf{r} = \begin{pmatrix} 4 \\ -5 \\ 6 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ 4 \\ -8 \end{pmatrix}$.

(a) Show that l_1 and l_2 are not parallel. [1 mark]

(b) Show that l_1 and l_2 intersect, and find the coordinates of the point of intersection. [4 marks]

(c) Find the acute angle between l_1 and l_2, giving your answer to the nearest $0.1°$. [3 marks]

(a) $\begin{pmatrix} 1 \\ 4 \\ -8 \end{pmatrix}$ is not a multiple of $\begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}$ (the x-components give ½, the y-components 4), so not parallel. **E1**

directions not multiples

(b) $2\lambda - \mu = 3$ (1), $\lambda - 4\mu = -2$ (2), $2 - 2\lambda = 6 - 8\mu$ (3) **M1** — components equal

(1) − 2 × (2): $7\mu = 7$, $\mu = 1$, $\lambda = 2$ **M1 A1** — solve (1) and (2)

(3): $2 - 4 = -2$, $6 - 8 = -2$ ✓, so they intersect at $(5, -1, -2)$ **A1** — check (3); substitute

(c) $\mathbf{d}_1 \cdot \mathbf{d}_2 = 2 + 4 + 16 = 22$ **M1** — scalar product of directions

$\cos\theta = \dfrac{22}{3 \times 9} = \dfrac{22}{27}$ **M1** — $|\mathbf{d}_1| = 3$, $|\mathbf{d}_2| = 9$

$\theta = 35.4°$ **A1** — inverse cos, 1 d.p.