Intersecting, Parallel or Skew Lines

Specification: AQA 7366/7367: F1 · Edexcel 9FM0: Core Pure 6.5

INDEPENDENT

You do

A · Lines that meet Write the three equations, solve two, check the third, then substitute.

1 l_1: $\mathbf{r} = \begin{pmatrix}1\\2\\0\end{pmatrix} + \lambda\begin{pmatrix}1\\1\\2\end{pmatrix}$ and l_2: $\mathbf{r} = \begin{pmatrix}5\\4\\3\end{pmatrix} + \mu\begin{pmatrix}3\\1\\1\end{pmatrix}$.
Show that they intersect, and find the point.

2 l_1: $\mathbf{r} = \begin{pmatrix}0\\3\\-1\end{pmatrix} + \lambda\begin{pmatrix}2\\-1\\1\end{pmatrix}$ and l_2: $\mathbf{r} = \begin{pmatrix}-4\\0\\4\end{pmatrix} + \mu\begin{pmatrix}1\\2\\-3\end{pmatrix}$.
Show that they intersect, and find the point.

3 l_1: $\mathbf{r} = (\mathbf{i} + \mathbf{k}) + \lambda(\mathbf{i} - \mathbf{j} + 2\mathbf{k})$ and
l_2: $\mathbf{r} = (3\mathbf{i} - 5\mathbf{j} + 2\mathbf{k}) + \mu(\mathbf{j} + \mathbf{k})$.
Show that they intersect, and find the point.

4 l_1: $\mathbf{r} = \begin{pmatrix}4\\-2\\1\end{pmatrix} + \lambda\begin{pmatrix}-1\\2\\3\end{pmatrix}$ and l_2: $\mathbf{r} = \begin{pmatrix}7\\2\\2\end{pmatrix} + \mu\begin{pmatrix}2\\1\\-1\end{pmatrix}$.
Show that they intersect, and find the point.

B · Meet or skew? The third equation decides.

5 l_1: $\mathbf{r} = \begin{pmatrix}1\\1\\1\end{pmatrix} + \lambda\begin{pmatrix}1\\2\\-1\end{pmatrix}$ and l_2: $\mathbf{r} = \begin{pmatrix}0\\2\\1\end{pmatrix} + \mu\begin{pmatrix}2\\1\\1\end{pmatrix}$.
Intersect or skew? If they meet, find the point.

6 l_1: $\mathbf{r} = \begin{pmatrix}2\\0\\-3\end{pmatrix} + \lambda\begin{pmatrix}1\\-2\\4\end{pmatrix}$ and l_2: $\mathbf{r} = \begin{pmatrix}-1\\0\\-3\end{pmatrix} + \mu\begin{pmatrix}1\\1\\-2\end{pmatrix}$.
Intersect or skew? If they meet, find the point.

7 l_1: $\mathbf{r} = \begin{pmatrix}0\\1\\2\end{pmatrix} + \lambda\begin{pmatrix}3\\0\\1\end{pmatrix}$ and l_2: $\mathbf{r} = \begin{pmatrix}4\\0\\2\end{pmatrix} + \mu\begin{pmatrix}1\\-1\\2\end{pmatrix}$.
Intersect or skew? If they meet, find the point.

8 l_1: $\mathbf{r} = \begin{pmatrix}1\\1\\0\end{pmatrix} + \lambda\begin{pmatrix}1\\-1\\1\end{pmatrix}$ and l_2: $\dfrac{x-1}{2} = \dfrac{y+4}{3} = 3 - z$.
Intersect or skew? If they meet, find the point.

C · Parallel, the same line, intersecting or skew Compare the directions first.

9 l_1: $\mathbf{r} = \begin{pmatrix}1\\2\\3\end{pmatrix} + \lambda\begin{pmatrix}2\\-1\\4\end{pmatrix}$ and l_2: $\mathbf{r} = \begin{pmatrix}3\\0\\5\end{pmatrix} + \mu\begin{pmatrix}-4\\2\\-8\end{pmatrix}$.
Parallel, the same line, intersecting or skew?

10 l_1: $\dfrac{x-1}{3} = \dfrac{y+2}{-1} = \dfrac{z-4}{2}$ and l_2: $\mathbf{r} = \begin{pmatrix}7\\-4\\8\end{pmatrix} + \mu\begin{pmatrix}6\\-2\\4\end{pmatrix}$.
Parallel, the same line, intersecting or skew?

11 l_1: $x - 2 = \dfrac{y-1}{2} = \dfrac{z+1}{-1}$ and l_2: $\dfrac{x-6}{3} = y - 4 = z - 1$.
Parallel, the same line, intersecting or skew?

12 l_1: $\mathbf{r} = (3\mathbf{i} - \mathbf{j} + 2\mathbf{k}) + \lambda(\mathbf{i} - 2\mathbf{k})$ and
l_2: $\mathbf{r} = (3\mathbf{i} - 3\mathbf{j} + 4\mathbf{k}) + \mu(-\mathbf{i} + \mathbf{j} + \mathbf{k})$.
Parallel, the same line, intersecting or skew?

D · Unknowns, angles and axes Solve the two equations without the unknown first.

13 l_1: $\mathbf{r} = \begin{pmatrix}1\\k\\2\end{pmatrix} + \lambda\begin{pmatrix}2\\1\\-1\end{pmatrix}$ and l_2: $\mathbf{r} = \begin{pmatrix}1\\2\\-1\end{pmatrix} + \mu\begin{pmatrix}1\\-1\\1\end{pmatrix}$ intersect.
Find k and the point.

14 l_1: $\mathbf{r} = \begin{pmatrix}0\\1\\-1\end{pmatrix} + \lambda\begin{pmatrix}1\\-2\\2\end{pmatrix}$ and l_2: $\mathbf{r} = \begin{pmatrix}3\\0\\-1\end{pmatrix} + \mu\begin{pmatrix}2\\1\\-2\end{pmatrix}$.
(a) Show that they intersect.
(b) Find the acute angle between them.

15 Show that the line $\mathbf{r} = \begin{pmatrix}2\\-4\\1\end{pmatrix} + \lambda\begin{pmatrix}1\\-2\\3\end{pmatrix}$ meets the z-axis, and find where.

16 l_1: $\dfrac{x-3}{2} = y - k = \dfrac{z+1}{-2}$ and l_2: $\mathbf{r} = \begin{pmatrix}3\\3\\3\end{pmatrix} + \mu\begin{pmatrix}1\\1\\1\end{pmatrix}$ intersect. Find k and the point.

E · Exam-style question Show your method: marks are for working.

17 The lines l_1 and l_2 have equations
l_1: $\mathbf{r} = \begin{pmatrix}1\\-3\\2\end{pmatrix} + \lambda\begin{pmatrix}2\\1\\-2\end{pmatrix}$ and l_2: $\mathbf{r} = \begin{pmatrix}4\\-5\\6\end{pmatrix} + \mu\begin{pmatrix}1\\4\\-8\end{pmatrix}$.
(a) Show that l_1 and l_2 are not parallel. [1 mark]
(b) Show that l_1 and l_2 intersect, and find the coordinates of the point of intersection. [4 marks]
(c) Find the acute angle between l_1 and l_2, giving your answer to the nearest 0.1°. [3 marks]