Shortest Distances

Specification: AQA 7366/7367: F1, F5 · Edexcel 9FM0: Core Pure 6.5

HOMEWORK Consolidation

1 Find the foot of the perpendicular from $\mathrm{P}(5, -1, 2)$ to l: $\mathbf{r} = \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix}$, and the shortest distance from P to l.

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2 Find the coordinates of the foot of the perpendicular from $\mathrm{P}(4, 4, -5)$ to the line l: $\mathbf{r} = (4\mathbf{i} + \mathbf{j} - 2\mathbf{k}) + \lambda(\mathbf{i} - \mathbf{j} + \mathbf{k})$.

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3 Find the shortest distance from $\mathrm{P}(6, 2, 3)$ to the line l: $x - 3 = \dfrac{y + 1}{3} = 2 - z$.

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4 Find the point of the line $\mathbf{r} = \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$ that is closest to the origin, and its distance from the origin.

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5 Show that l_1: $\mathbf{r} = \begin{pmatrix} 1 \\ 4 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix}$ and l_2: $\mathbf{r} = \begin{pmatrix} -1 \\ 2 \\ -3 \end{pmatrix} + \mu \begin{pmatrix} -6 \\ -2 \\ -4 \end{pmatrix}$ are parallel, and find the distance between them.

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6 Show that l_1: $\dfrac{x + 2}{2} = 1 - y = \dfrac{z - 5}{2}$ and l_2: $\mathbf{r} = \begin{pmatrix} 1 \\ 2 \\ 7 \end{pmatrix} + \mu \begin{pmatrix} 4 \\ -2 \\ 4 \end{pmatrix}$ are parallel, and find the distance between them.

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7 Find the shortest distance between l_1: $\mathbf{r} = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$ and l_2: $\mathbf{r} = \begin{pmatrix} 1 \\ -4 \\ -1 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}$.

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8 Find the points P on l_1 and Q on l_2 that are closest together, and the shortest distance, for l_1: $\mathbf{r} = \begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$ and l_2: $\mathbf{r} = \begin{pmatrix} 0 \\ 3 \\ 5 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$.

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9 Find the shortest distance between l_1: $\mathbf{r} = \begin{pmatrix} 2 \\ -1 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$ and l_2: $\mathbf{r} = \begin{pmatrix} 1 \\ 0 \\ -5 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}$. Interpret your answer.

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10 The line l passes through $\mathrm{A}(1, 2, 0)$ and $\mathrm{B}(3, 1, 2)$. The point P has coordinates $(6, 0, 3)$.
(a) Find the shortest distance from P to l.
(b) Hence find the exact area of the triangle ABP.

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11 *AQA-style*. The points A and B have coordinates $(1, 0, -2)$ and $(3, 1, -4)$ respectively, and C has coordinates $(6, 2, -5)$.
(a) Find a vector equation of the line AB. [2 marks]
(b) Find the coordinates of the point M on the line AB that is closest to C. [4 marks]
(c) Find the exact shortest distance from C to the line AB. [1 mark]

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12 *Edexcel-style*. The lines l_1 and l_2 have equations
l_1: $\mathbf{r} = (-\mathbf{i} + 2\mathbf{j}) + \lambda(\mathbf{i} + 2\mathbf{j} - \mathbf{k})$
l_2: $\mathbf{r} = (3\mathbf{i} - 5\mathbf{k}) + \mu(2\mathbf{i} - \mathbf{j} + \mathbf{k})$
(a) Find the shortest distance between l_1 and l_2, giving your answer to 3 significant figures. (5)
(b) Hence explain why l_1 and l_2 do not intersect. (1)

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