Shortest Distances

Specification: AQA 7366/7367: F1, F5 · Edexcel 9FM0: Core Pure 6.5

INDEPENDENT **You do**

A · Point to a line M on the line, PM·d = 0, then |PM|. Exact answers.

1 Find the foot of the perpendicular from P(4, 2, 1) to l: $\mathbf{r} = \begin{pmatrix}1\\1\\0\end{pmatrix} + \lambda\begin{pmatrix}1\\0\\1\end{pmatrix}$, and the shortest distance from P to l.

2 Find the foot of the perpendicular from P(3, −2, 7) to l: $\mathbf{r} = \begin{pmatrix}2\\-1\\3\end{pmatrix} + \lambda\begin{pmatrix}1\\2\\-2\end{pmatrix}$, and the shortest distance from P to l.

3 Find the foot of the perpendicular from P(4, 6, 0) to l: $\mathbf{r} = \begin{pmatrix}0\\4\\-2\end{pmatrix} + \lambda\begin{pmatrix}3\\0\\-1\end{pmatrix}$, and the shortest distance from P to l.

4 Find the foot of the perpendicular from P(−2, −8, 0) to l: $\mathbf{r} = (\mathbf{i} - 2\mathbf{j}) + \lambda(2\mathbf{i} + 2\mathbf{j} + \mathbf{k})$, and the shortest distance from P to l.

B · Other forms, the origin and reflections Find a point and a direction first.

5 The line l passes through A(2, 1, −1) and B(4, 2, 1). Find the shortest distance from P(1, 0, −4) to l.

6 Find the shortest distance from P(2, 3, 7) to the line l: $\frac{x+1}{2} = \frac{3-y}{2} = z - 4$.

7 Find the point of the line $\mathbf{r} = \begin{pmatrix}0\\-5\\-4\end{pmatrix} + \lambda\begin{pmatrix}1\\2\\2\end{pmatrix}$ that is closest to the origin O, and its distance from O.

8 Find the image of P(3, 1, 5) after reflection in the line l: $\mathbf{r} = \begin{pmatrix}1\\0\\3\end{pmatrix} + \lambda\begin{pmatrix}1\\-1\\1\end{pmatrix}$.

C · Parallel lines Any point of one line; its distance to the other line.

9 Show that l_1: $\mathbf{r} = \begin{pmatrix}1\\1\\1\end{pmatrix} + \lambda\begin{pmatrix}1\\2\\-1\end{pmatrix}$ and l_2: $\mathbf{r} = \begin{pmatrix}3\\3\\1\end{pmatrix} + \mu\begin{pmatrix}2\\4\\-2\end{pmatrix}$ are parallel, and find the distance between them.

10 Show that l_1: $\mathbf{r} = (3\mathbf{i} - \mathbf{k}) + \lambda(2\mathbf{i} - 2\mathbf{j} + \mathbf{k})$ and l_2: $\mathbf{r} = (3\mathbf{i} + 3\mathbf{j} - 4\mathbf{k}) + \mu(-2\mathbf{i} + 2\mathbf{j} - \mathbf{k})$ are parallel, and find the distance between them.

11 Show that l_1: $\frac{x-1}{2} = y + 1 = \frac{z-2}{-2}$ and l_2: $\mathbf{r} = \begin{pmatrix}8\\-1\\0\end{pmatrix} + \mu\begin{pmatrix}4\\2\\-4\end{pmatrix}$ are parallel, and find the distance between them.

D · Skew lines P on l_1 and Q on l_2 both move: PQ·$\mathbf{d}_1$ = 0 and PQ·$\mathbf{d}_2$ = 0.

12 Find the shortest distance between l_1: $\mathbf{r} = \begin{pmatrix}2\\0\\1\end{pmatrix} + \lambda\begin{pmatrix}1\\-1\\2\end{pmatrix}$ and l_2: $\mathbf{r} = \begin{pmatrix}3\\1\\4\end{pmatrix} + \mu\begin{pmatrix}1\\1\\0\end{pmatrix}$.

13 Find the shortest distance between l_1: $\mathbf{r} = \begin{pmatrix}0\\1\\-2\end{pmatrix} + \lambda\begin{pmatrix}2\\1\\0\end{pmatrix}$ and l_2: $\mathbf{r} = \begin{pmatrix}2\\1\\-6\end{pmatrix} + \mu\begin{pmatrix}1\\-1\\1\end{pmatrix}$.

14 Find the points P on l_1 and Q on l_2 that are closest together, and the shortest distance, for l_1: $\mathbf{r} = \begin{pmatrix}3\\-1\\0\end{pmatrix} + \lambda\begin{pmatrix}1\\2\\-2\end{pmatrix}$ and l_2: $\mathbf{r} = \begin{pmatrix}4\\-9\\-2\end{pmatrix} + \mu\begin{pmatrix}2\\1\\1\end{pmatrix}$.

15 Find the shortest distance between l_1: $\mathbf{r} = \begin{pmatrix}1\\2\\-1\end{pmatrix} + \lambda\begin{pmatrix}1\\0\\2\end{pmatrix}$ and l_2: $\mathbf{r} = \begin{pmatrix}5\\3\\2\end{pmatrix} + \mu\begin{pmatrix}2\\1\\-1\end{pmatrix}$. What does your answer tell you?

E · Exam-style question Show your method: marks are for working.

16 The lines l_1 and l_2 have equations l_1: $\mathbf{r} = \begin{pmatrix}1\\1\\-2\end{pmatrix} + \lambda\begin{pmatrix}1\\-1\\2\end{pmatrix}$ and l_2: $\mathbf{r} = \begin{pmatrix}-4\\2\\3\end{pmatrix} + \mu\begin{pmatrix}2\\1\\0\end{pmatrix}$.

(a) Show that l_1 and l_2 are not parallel. [1 mark]

(b) The point P lies on l_1 and the point Q lies on l_2, and PQ is perpendicular to both lines. Find the coordinates of P and Q. [5 marks]

(c) Hence find the exact shortest distance between l_1 and l_2. [2 marks]