Work Done and Kinetic Energy

Specification: AQA 7367: MC1, MC3 · Edexcel 9FM0: FM1 Work, energy and power

INDEPENDENT

You do

A · Work done

Forces along the motion, then at an angle.

1 A horizontal force of 25 N pulls a box 8 m along a horizontal floor. Find the work done by the force.

$W = Fd$ — W = Fd

$W = 25 \times 8 = 200$ J — substitute

2 A train travels 0.4 km against a constant resistance of 12 kN. Find the work done against the resistance.

0.4 km = 400 m, 12 kN = 12 000 N — convert units

$W = 12\,000 \times 400 = 4\,800\,000$ J — W = Fd

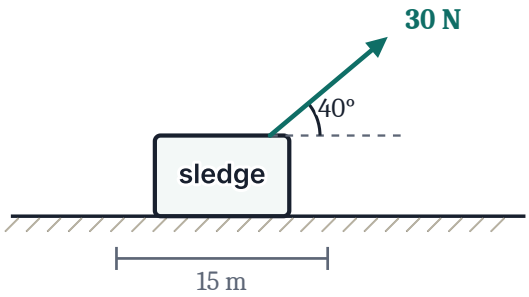
$W = 4.8$ MJ — in MJ

3 A constant force does 630 J of work moving an object 14 m in the direction of the force. Find the force.

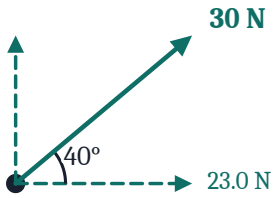
$W = Fd$: $630 = 14F$ — W = Fd

$F = 45$ N — ÷ 14

4 A rope with tension 30 N at 40° to the horizontal pulls a sledge 15 m across horizontal ground. Find the work done by the tension.



Component along the motion: $30 \cos 40° = 22.98\ldots$ N — component along motion



picture: only 30 cos 40° does work

$W = 30 \cos 40° \times 15$ — W = Fd cos θ

$W = 345$ J — 3 s.f.

B · Kinetic energy

Convert to kg and m s^{-1} first.

5 Find the kinetic energy of a ball of mass 3 kg moving at 4 m s^{-1}.

$KE = \frac{1}{2} \times 3 \times 4^2$ — KE = ½mv²

$KE = 24$ J — evaluate

6 Find the kinetic energy of a car of mass 1.2 tonnes travelling at 90 km h^{-1}.

1200 kg, $90 \div 3.6 = 25$ m s^{-1} — convert units

$KE = \frac{1}{2} \times 1200 \times 25^2$ — KE = ½mv²

$KE = 375\,000$ J — evaluate

7 A ball of mass 0.2 kg has 14.4 J of kinetic energy. Find its speed.

$\frac{1}{2} \times 0.2 \times v^2 = 14.4$ — KE = ½mv²

$0.1v^2 = 14.4$, $v^2 = 144$ — ÷ 0.1

$v = 12$ m s^{-1} — square root

8 A particle moving at 6 m s^{-1} has 54 J of kinetic energy. Find its mass.

$\frac{1}{2} \times m \times 6^2 = 54$ — KE = ½mv²

$18m = 54$ — ½ × 36 = 18

$m = 3$ kg — ÷ 18

C · Reasoning (no calculator) Explain in words as well as numbers.

9 The speed of a body doubles. What happens to its kinetic energy? Explain.

New KE = $\frac{1}{2}m(2v)^2$ — replace v by 2v

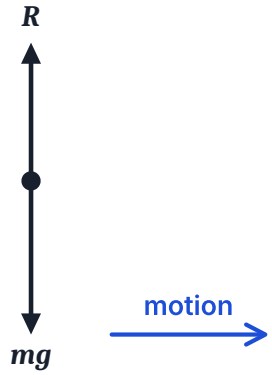
$\frac{1}{2}m(2v)^2 = 4 \times \frac{1}{2}mv^2$ — speed is squared

The kinetic energy is multiplied by 4. — ×4, not ×2

10 A box slides along a horizontal floor. Explain why the normal reaction does no work on the box.

The normal reaction is vertical; the box moves horizontally.

perpendicular to motion

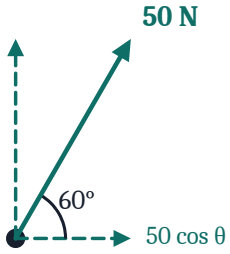


picture: R ⊥ the motion

$W = Fd\cos 90° = 0$ — cos 90° = 0

11 A force of 50 N pulls a box 10 m along a horizontal floor. The force does 250 J of work. Find the angle between the force and the floor.

$W = Fd\cos\theta$: $50 \times 10 \times \cos\theta = 250$ — W = Fd cos θ



picture: only 50 cos θ does work

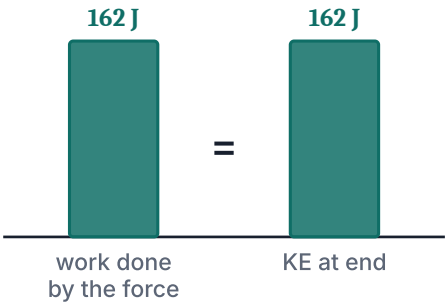
$\cos\theta = 250/500 = 0.5$ — ÷ 500

$\theta = 60°$ — exact value

D · Work–energy principle Total work done = change in kinetic energy. Take g = 9.8 m s^{-2}.

12 A particle of mass 4 kg is at rest on a smooth horizontal surface. A horizontal force of 18 N acts on it while it moves 9 m. Find its final speed.

Work done = $18 \times 9 = 162$ J — W = Fd



picture: energy

$\frac{1}{2} \times 4 \times v^2 = 162$ — work = gain in KE

$v^2 = 81$ — ÷ 2

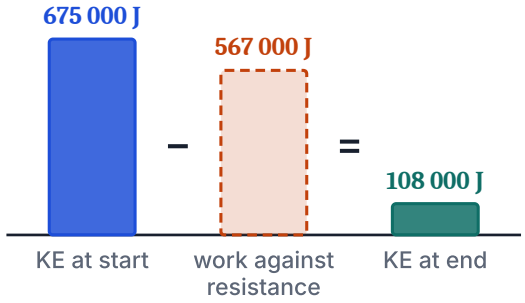
$v = 9$ m s^{-1} — square root

13 A car of mass 1500 kg slows from 30 m s^{-1} to 12 m s^{-1} over 120 m on a horizontal road. Find the constant braking force.

KE at start = $\frac{1}{2} \times 1500 \times 30^2 = 675\,000$ J — KE = ½mv²

KE at end = $\frac{1}{2} \times 1500 \times 12^2 = 108\,000$ J — KE = ½mv²

Loss in KE = $675\,000 - 108\,000 = 567\,000$ J — loss in KE

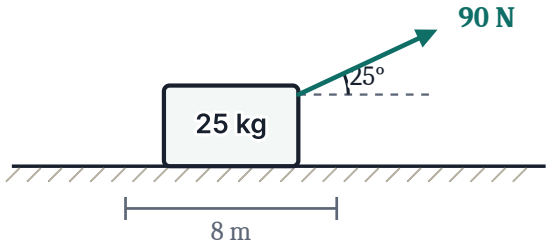


picture: energy

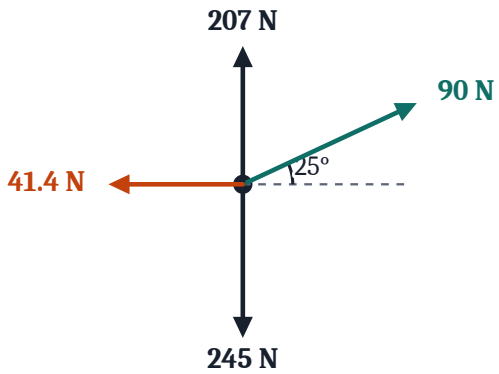
$120F = 567\,000$ — work against braking = KE lost

$F = 4725$ N — ÷ 120

14 A box of mass 25 kg is pulled from rest 8 m across a rough horizontal floor by a rope at 25° above the horizontal. The tension is 90 N and the coefficient of friction is 0.2. Take $g = 9.8$ m s^{-2}. Find the speed of the box.



Resolve ↑: $R = 25g - 90 \sin 25° = 245 - 38.03... = 206.96...$ N — resolve ↑



picture: forces

$F = 0.2 \times 206.96... = 41.39...$ N — F = μR

Work by tension = $90 \cos 25° \times 8 = 652.54...$ J — W = Fd cos θ

Work against friction = $41.39... \times 8 = 331.14...$ J — W = Fd

$\frac{1}{2} \times 25 \times v^2 = 652.54... - 331.14... = 321.39...$ — total work = gain in KE

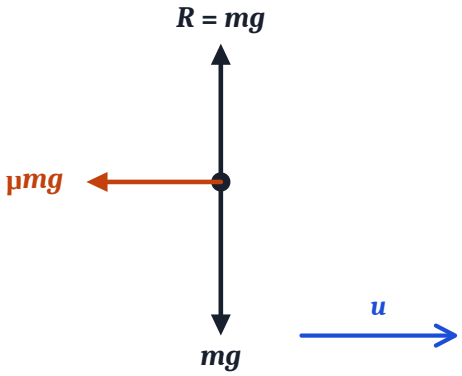
$v^2 = 25.71...$ — ÷ 12.5

$v = 5.07$ m s^{-1} — square root, 3 s.f.

15 A car travelling at speed u brakes on a horizontal road with its wheels locked. The coefficient of friction between the tyres and the road is μ.
(a) Show that the car stops in a distance $u^2/(2\mu g)$.
(b) Explain why this distance does not depend on the mass of the car.

(a) Resolve ↑: $R = mg$ — resolve ↑

Friction: $F = \mu R = \mu mg$ — F = μR



picture: friction μmg

Work against friction over d: μmgd — W = Fd

$\mu mgd = \frac{1}{2}mu^2 - 0$ — work against friction = KE lost

$d = u^2/(2\mu g)$ — ÷ μmg

(b) Both the friction and the KE are proportional to m, so m cancels.

m cancels

E · Exam-style question Show your method: marks are for working.

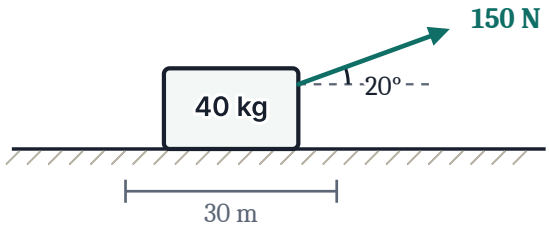
16 A sled of mass 40 kg is pulled from rest across rough horizontal snow by a rope at 20° above the horizontal. The tension is 150 N and the coefficient of friction is 0.15. Take $g = 9.8$ m s^{-2}.

(a) Show that the normal reaction is 341 N to 3 s.f. [2]

(b) Find the work done against friction over 30 m. [2]

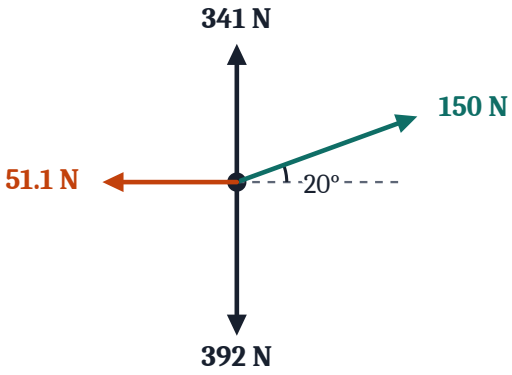
(c) Use the work–energy principle to find the speed after 30 m. [3]

(d) State one modelling assumption. [1]



(a) Resolve ↑: $R + 150 \sin 20° = 40g$ **M1** — resolve ↑

$R = 392 - 51.30... = 340.69... = 341$ N **A1** — 3 s.f.



picture: forces

(b) $F = \mu R = 0.15 \times 340.69... = 51.10...$ N **M1** — F = μR

Work against friction $= 51.10... \times 30 = 1533.1... = 1530$ J (3 s.f.) **A1** — W = Fd

(c) Work by the tension $= 150 \cos 20° \times 30 = 4228.6...$ J **M1** — W = Fd cos θ

$\frac{1}{2} \times 40 \times v^2 = 4228.6... - 1533.1... = 2695.5...$ **M1** — total work = gain in KE

$v^2 = 134.77...$ — ÷ 20

$v = 11.6$ m s^{-1} **A1** — square root, 3 s.f.

(d) The sled is a particle (or: the rope is light and stays at 20°; the coefficient of friction is constant). **B1** — any one