GPE and Conservation of Mechanical Energy

Specification: AQA 7367: MC1, MC2, MC3 · Edexcel 9FM0: FM1 Work, energy and power

INDEPENDENT ## You do

A · Gravitational potential energy GPE = mgh, with h measured from the zero level.

1 A diver of mass 70 kg stands on a board 10 m above the water. Taking the water surface as the zero level, find the GPE of the diver. Take $g = 9.8$ m s^{-2}.

GPE $= mgh = 70 \times 9.8 \times 10$ — GPE = mgh

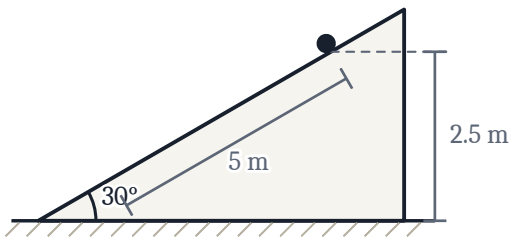
GPE = 6860 J — evaluate

2 A lift raises a load of mass 600 kg through a height of 25 m. Find the gain in GPE of the load. Take $g = 9.8$ m s^{-2}.

Gain $= 600 \times 9.8 \times 25 = 147\,000$ J — GPE = mgh

$= 147$ kJ — in kJ

3 A particle of mass 2 kg moves 5 m up a line of greatest slope of a plane inclined at 30° to the horizontal. Find its gain in GPE. Take $g = 9.8$ m s^{-2}.



picture: h = 5 sin 30°

Vertical height gained $= 5 \sin 30° = 2.5$ m — vertical height: d sin θ

Gain in GPE $= 2 \times 9.8 \times 2.5 = 49$ J — GPE = mgh

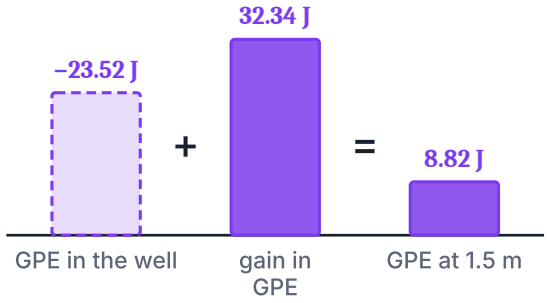
4 A ball of mass 0.6 kg is at the bottom of a well, 4 m below ground level. Take ground level as the zero level. Take $g = 9.8$ m s^{-2}.
(a) Find the GPE of the ball.
(b) The ball is lifted to 1.5 m above ground level. Find its gain in GPE.

(a) Below the zero level: $h = -4$ — below: h negative

GPE $= 0.6 \times 9.8 \times (-4) = -23.52$ J — GPE = mgh

(b) At 1.5 m above ground: GPE $= 0.6 \times 9.8 \times 1.5 = 8.82$ J — GPE = mgh

Gain $= 8.82 - (-23.52) = 32.34$ J — final − initial



picture: GPE (ground = 0)

Check: it rises $4 + 1.5 = 5.5$ m, and $0.6 \times 9.8 \times 5.5 = 32.34$ — check

B · Conservation of energy

Only the weight does work: GPE lost = KE gained.

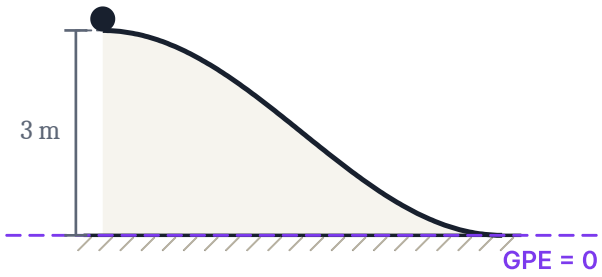
5 A ball is dropped from rest from a height of 7.2 m. Use conservation of energy to find its speed as it hits the ground. Take $g = 9.8$ m s^{-2}.

$\frac{1}{2}mv^2 = mgh$ — GPE lost = KE gained

$v^2 = 2gh = 2 \times 9.8 \times 7.2 = 141.12$ — ÷ m, × 2

$v = 11.9$ m s^{-1} — square root, 3 s.f.

6 A particle is released from rest on a smooth curved slide, 3 m vertically above the bottom of the slide. Find its speed at the bottom. Take $g = 9.8$ m s^{-2}.



picture: a fall of 3 m

The normal reaction does no work, so GPE lost = KE gained: $\frac{1}{2}mv^2 = mg \times 3$

only the weight does work

$v^2 = 2 \times 9.8 \times 3 = 58.8$ — ÷ m, × 2

$v = 7.67$ m s^{-1} — square root, 3 s.f.

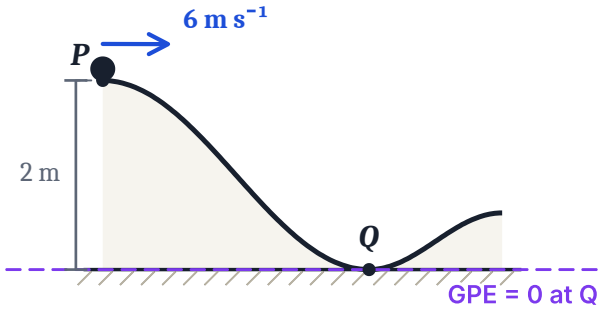
7 A ball is thrown vertically upwards at 15 m s^{-1}. Use conservation of energy to find the greatest height it reaches above the point of projection. Take $g = 9.8$ m s^{-2}.

At the top $v = 0$: $\frac{1}{2}mu^2 = mgh$ — KE lost = GPE gained

$h = 15^2/(2 \times 9.8) = 11.47\ldots$ — ÷ mg

$h = 11.5$ m — 3 s.f.

8 A bead slides along a smooth wire. It passes point P at 6 m s^{-1}. Point Q is 2 m below P. Find the speed of the bead at Q. Take $g = 9.8$ m s^{-2}.



picture: P is 2 m above Q

$\frac{1}{2}mv^2 = \frac{1}{2}m \times 6^2 + mg \times 2$ — KE + GPE constant

$v^2 = 36 + 2 \times 9.8 \times 2 = 75.2$ — ÷ m, × 2

$v = 8.67$ m s^{-1} — square root, 3 s.f.

C · Reasoning (no calculator)

Explain in words as well as numbers.

9 Two particles, of mass 1 kg and 3 kg, are released from rest at the same point on a smooth curved track. Compare their speeds when they reach the bottom. Explain.

$mgh = \frac{1}{2}mv^2$ for each particle — GPE lost = KE gained

$v^2 = 2gh$: m cancels — ÷ m

The speeds are equal. The 3 kg particle has 3 times the KE, but it also had 3 times the GPE to lose.

same speed

10 A stone is thrown from the top of a cliff at speed u, first horizontally and then at 40° above the horizontal. Ignoring air resistance, explain why it hits the sea at the same speed both times.

$\frac{1}{2}mv^2 = \frac{1}{2}mu^2 + mgh$ both times — conservation of energy

KE depends on the speed only, not the direction, and u and h are the same both times.

energy has no direction

So $v^2 = u^2 + 2gh$ both times: the same speed, but a different direction.

same speed

11 A particle on a smooth track passes its lowest point at 7 m s^{-1}. Find the greatest height above the lowest point that it can reach. Take $g = 9.8$ m s^{-2}.

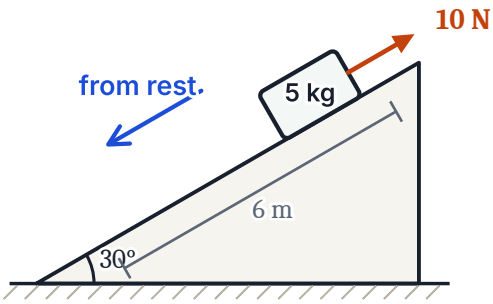
$\frac{1}{2}mv^2 = mgh$ — KE lost = GPE gained

$h = v^2/(2g) = 49/19.6$ — ÷ mg

$h = 2.5$ m — 49 ÷ 19.6

D · Other forces, projectiles and a show-that Work done by other forces = change in KE + change in GPE.

12 A box of mass 5 kg slides from rest 6 m down a slope inclined at 30° to the horizontal. A constant resistance of 10 N acts on the box. Find the speed of the box after 6 m. Take $g = 9.8$ m s^{-2}.

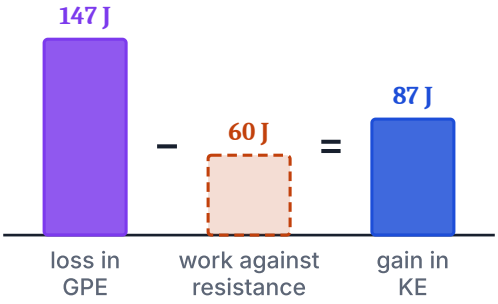


Height lost = $6 \sin 30° = 3$ m — vertical height

GPE lost = $5 \times 9.8 \times 3 = 147$ J — GPE = mgh

Work against the resistance = $10 \times 6 = 60$ J — W = Fd

Gain in KE = $147 - 60 = 87$ J — GPE lost − work against

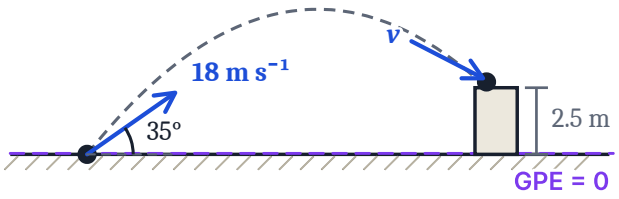


picture: energy

$\frac{1}{2} \times 5 \times v^2 = 87$, $v^2 = 34.8$ — KE = ½mv²

$v = 5.90$ m s^{-1} — square root, 3 s.f.

13 A ball is kicked from the ground at 18 m s^{-1} at 35° above the horizontal. It lands on the top of a wall 2.5 m high. Ignore air resistance. Take $g = 9.8$ m s^{-2}.
(a) Use energy to find its speed as it lands on the wall.
(b) Explain why the angle 35° is not needed in (a).



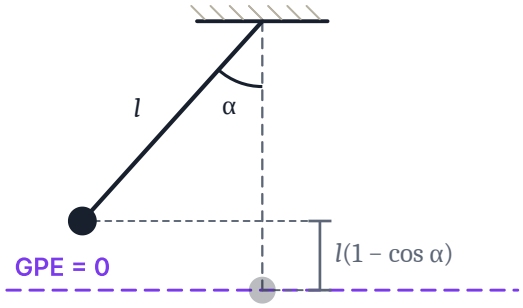
picture: the wall

(a) $\frac{1}{2}mv^2 + mg \times 2.5 = \frac{1}{2}m \times 18^2$ — KE + GPE constant
$\frac{1}{2}v^2 = 162 - 24.5 = 137.5$ — ÷ m
$v^2 = 275$ — × 2
$v = 16.6$ m s^{-1} — square root, 3 s.f.

(b) KE depends only on the speed, and the change in GPE depends only on the change in height, not on the path. So the direction is not needed.

energy: no direction

14 A particle hangs on a light inextensible string of length l. It is released from rest with the string taut at an angle α to the vertical.
(a) Show that its speed v at the lowest point is given by $v^2 = 2gl(1 - \cos\alpha)$.
(b) The string is 0.8 m long and $v = 2$ m s^{-1}. Find α. Take $g = 9.8$ m s^{-2}.



picture: h = l(1 − cos α)

(a) Height above the lowest point:
$h = l - l\cos\alpha = l(1 - \cos\alpha)$ — pendulum height
The tension is perpendicular to the motion: it does no work. — tension: no work
$\frac{1}{2}mv^2 = mgl(1 - \cos\alpha)$ — KE gained = GPE lost
$v^2 = 2gl(1 - \cos\alpha)$ — ÷ m, × 2

(b) $2gl = 2 \times 9.8 \times 0.8 = 15.68$, so $15.68(1 - \cos\alpha) = 2^2$ — substitute
$1 - \cos\alpha = 4/15.68 = 0.2551...$ — ÷ 15.68
$\cos\alpha = 0.7448...$ — rearrange
$\alpha = 41.8°$ — inverse cos, 3 s.f.

15 A car of mass 900 kg travels 150 m up a straight road inclined at 4° to the horizontal. Its speed falls from 20 m s^{-1} to 15 m s^{-1}. The resistance to motion is a constant 400 N and the driving force is a constant D N. Find D. Take $g = 9.8$ m s^{-2}.

Change in KE = ½ × 900 × ($15^2 - 20^2$) = −78 750 J (a loss of 78 750 J)

change in KE

Gain in GPE = 900 × 9.8 × 150 sin 4° = 92 287.8... J

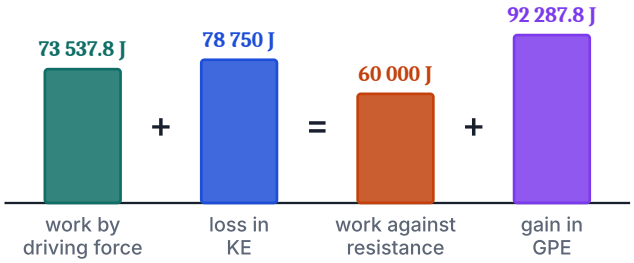
h = 150 sin 4°

Work by the other forces
= $150D - 400 \times 150 = 150D - 60\,000$

work by D − work against R

$150D - 60\,000 = -78\,750 + 92\,287.8...$ work = ΔKE + ΔGPE

$150D = 73\,537.8...$ + 60 000



picture: energy in = energy out

$D = 490$ N (3 s.f.) ÷ 150

E · Exam-style question Show your method: marks are for working.

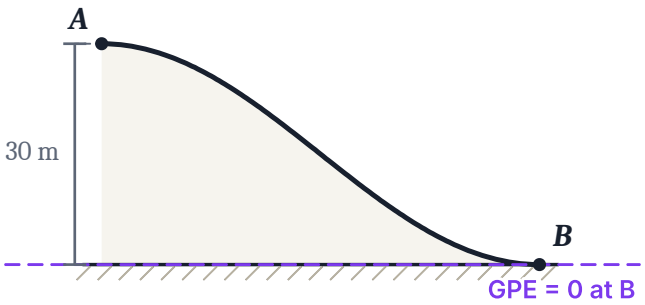
16 A skier of mass 70 kg starts from rest at A and skis down a slope to B, 30 m vertically below A. Take $g = 9.8$ m s^{-2}.

(a) Modelling the slope as smooth, use conservation of energy to find the speed at B. [3]

(b) In fact the skier reaches B at 18 m s^{-1}. Find the work done against the resistances. [3]

(c) The path from A to B is 120 m long. Find the mean resistance. [1]

(d) State one modelling assumption. [1]



(a) GPE = 0 at B. GPE lost = $70 \times 9.8 \times 30 = 20\,580$ J **M1** — GPE = mgh

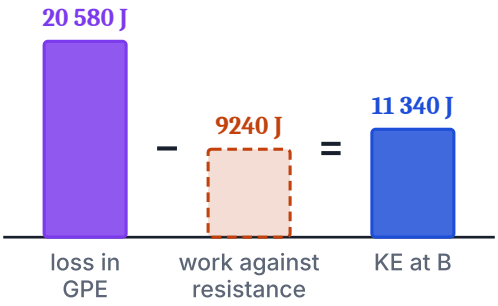
$\frac{1}{2} \times 70 \times v^2 = 20\,580$ **M1** — GPE lost = KE gained

$v^2 = 588$, $v = 24.2$ m s^{-1} **A1** — square root, 3 s.f.

(b) KE at $B = \frac{1}{2} \times 70 \times 18^2 = 11\,340$ J **M1** — KE = ½mv²

Work against resistances = GPE lost − KE gained = $20\,580 - 11\,340$ **M1** — work = energy lost

$= 9240$ J **A1** — evaluate



picture: energy

(c) $120R = 9240$, so $R = 77$ N **B1** — W = Fd

(d) The skier is a particle (or: air resistance and friction are combined as one resistance). **B1** — any one