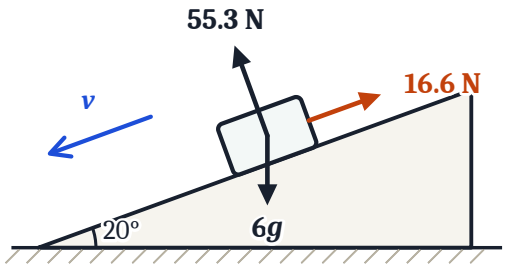
Work against Resistance: the Energy Audit

Specification: AQA 7367: MC1, MC2, MC3 · Edexcel 9FM0: FM1 Work, energy and power

INDEPENDENT ## You do

A · Work against resistance R = mg cos θ on a slope; F = μR; work = Fd.

1 A box of mass 6 kg slides 3 m down a rough plane inclined at 20° to the horizontal. The coefficient of friction is 0.3. Take $g = 9.8$ m s^{-2}.
(a) Find the normal reaction.
(b) Find the friction force.
(c) Find the work done against friction.



picture: forces

(a) $R = 6 \times 9.8 \times \cos 20° = 55.25... = 55.3$ N — R = mg cos θ

(b) $F = 0.3 \times 55.25... = 16.57... = 16.6$ N — F = μR

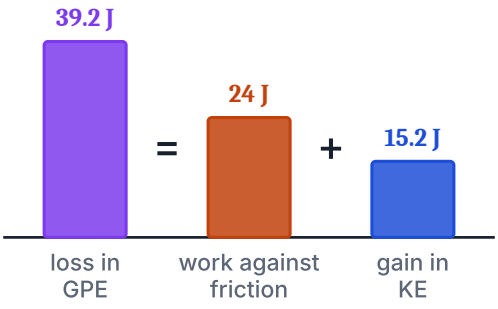
(c) Work = $16.57... \times 3 = 49.72... = 49.7$ J — W = Fd

2 A car of mass 1200 kg brakes on a level road. Its speed falls from 25 m s^{-1} to 10 m s^{-1} while it travels 80 m. Find the constant braking force.

KE lost = ½ × 1200 × ($25^2 - 10^2$) = 315 000 J — KE lost
Work against braking = KE lost: $80F = 315\,000$ — work = KE lost
$F = 3937.5 = 3940$ N (3 s.f.) — ÷ 80

3 A particle of mass 2 kg slides from rest 4 m down a rough slope inclined at 30° to the horizontal. A constant friction force of 6 N acts on it. Take $g = 9.8$ m s^{-2}. Find its speed after 4 m.

GPE lost = $2 \times 9.8 \times 4 \sin 30° = 39.2$ J — h = d sin θ
Work against friction = $6 \times 4 = 24$ J — W = Fd

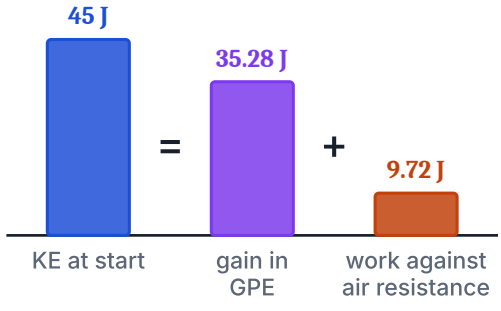


picture: energy audit

KE = 39.2 − 24 = 15.2 J — energy audit
$\frac{1}{2} \times 2 \times v^2 = 15.2$, $v^2 = 15.2$ — ½ × 2 = 1
$v = 3.90$ m s^{-1} — square root, 3 s.f.

4 A ball of mass 0.4 kg is thrown vertically upwards at 15 m s^{-1}. Because of air resistance it rises only 9 m. Take $g = 9.8$ m s^{-2}. Find the work done against air resistance.

KE lost = ½ × 0.4 × 15^2 = 45 J — KE = ½mv²
GPE gained = 0.4 × 9.8 × 9 = 35.28 J — GPE = mgh



picture: energy audit

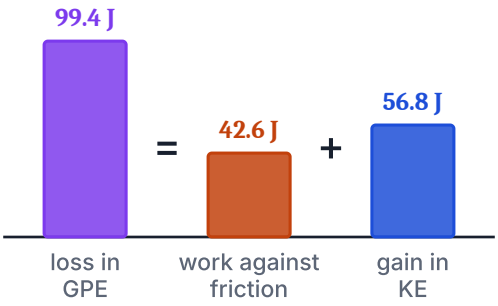
Work against air resistance = 45 − 35.28 = 9.72 J — energy audit

B · Rough slopes Energy at start − work against friction = energy at end.

5 A particle of mass 3 kg is released from rest on a rough plane inclined at 25° to the horizontal. The coefficient of friction is 0.2. Take $g = 9.8$ m s^{-2}. Find its speed after it has slid 8 m.

$R = 3 \times 9.8 \times \cos 25° = 26.64...$ N,
$F = 0.2 \times 26.64... = 5.329...$ N
R = mg cos θ, F = μR

Work against friction $= 5.329... \times 8 = 42.63...$ J — W = Fd
GPE lost $= 3 \times 9.8 \times 8 \sin 25° = 99.39...$ J — h = d sin θ
KE $= 99.39... - 42.63... = 56.76...$ J — energy audit

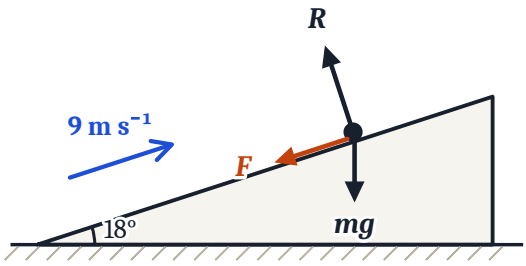


picture: energy audit

$\frac{1}{2} \times 3 \times v^2 = 56.76...$, $v^2 = 37.84...$ — KE = ½mv²

$v = 6.15$ m s^{-1} — square root, 3 s.f.

6 A particle is projected at 9 m s^{-1} up a line of greatest slope of a rough plane inclined at 18° to the horizontal. The coefficient of friction is 0.3. Take $g = 9.8$ m s^{-2}. Find the distance it travels before it stops.



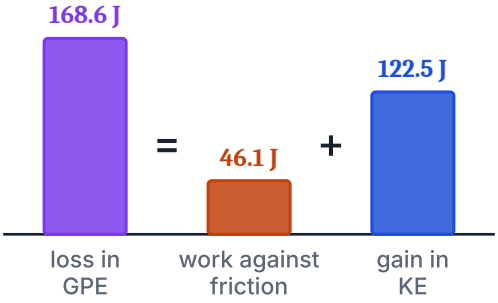
picture: friction acts down the slope

Mass m: $\frac{1}{2}mu^2 = mgs \sin 18° + 0.3mg \cos 18° \times s$ — KE = GPE + work against
$40.5 = s(9.8 \sin 18° + 0.3 \times 9.8 \cos 18°)$ — ÷ m
$40.5 = s(3.028... + 2.796...) = 5.824...s$ — evaluate

$s = 6.95$ m — ÷ 5.824...

7 A crate of mass 5 kg is released from rest on a rough plane inclined at 35° to the horizontal. After sliding 6 m its speed is 7 m s^{-1}. Take $g = 9.8$ m s^{-2}. Find the coefficient of friction.

GPE lost $= 5 \times 9.8 \times 6 \sin 35° = 168.6...$ J — h = d sin θ
KE gained $= \frac{1}{2} \times 5 \times 7^2 = 122.5$ J — KE = ½mv²
Work against friction $= 168.6... - 122.5 = 46.13...$ J — energy audit



picture: energy audit

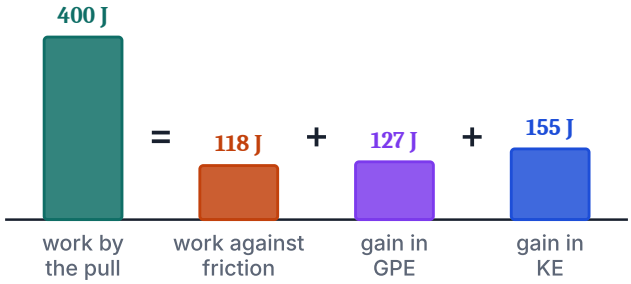
$R = 5 \times 9.8 \times \cos 35° = 40.13...$ N — R = mg cos θ
$\mu \times 40.13... \times 6 = 46.13...$, $240.8...\mu = 46.13...$ — work = μR × d

$\mu = 0.192$ — ÷ 240.8...

8 A box of mass 10 kg starts from rest and is pulled 5 m up a rough plane inclined at 15° to the horizontal by a force of 80 N parallel to the plane. The coefficient of friction is 0.25. Take $g = 9.8$ m s^{-2}. Find the speed of the box after 5 m.

Work by the pull $= 80 \times 5 = 400$ J — W = Fd
$R = 10 \times 9.8 \times \cos 15° = 94.66...$ N, work against friction $= 0.25 \times 94.66... \times 5 = 118.3...$ J — F = μR, W = Fd

GPE gained $= 10 \times 9.8 \times 5 \sin 15° = 126.8...$ J — h = d sin θ



picture: energy in = energy out

KE $= 400 - 118.3... - 126.8... = 154.8...$ J — energy audit
$\frac{1}{2} \times 10 \times v^2 = 154.8...$, $v^2 = 30.97...$ — KE = ½mv²

$v = 5.57$ m s^{-1} — square root, 3 s.f.

C · Reasoning (no calculator) Explain in words as well as numbers.

9 A puck is projected up a rough slope. It stops, then slides back. Explain why it returns to its starting point more slowly than it left.

The GPE it gains going up is all given back on the way down: no change in GPE overall.

GPE: back to the start

Friction acts against the motion both ways, so work is done against friction going up **and** coming down.

friction: both ways

KE at return = KE at start − work against friction (both ways), which is less: it is slower.

energy audit

10 A block is placed at rest on a rough plane inclined at θ to the horizontal. The coefficient of friction is μ. Use the energy for each metre down the plane to explain why the block slides only if $\mu < \tan\theta$.

Each metre down: GPE lost = $mg\sin\theta$, work against friction = $\mu mg\cos\theta$

per metre

It gains KE only if $mg\sin\theta > \mu mg\cos\theta$

energy left for KE

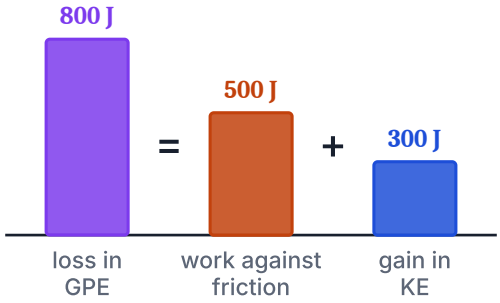
Divide by $mg\cos\theta$: $\tan\theta > \mu$

÷ mg cos θ

11 A sledge slides 10 m down a slope. It loses 800 J of GPE and gains 300 J of KE.

(a) Find the work done against friction.

(b) Find the friction force, assumed constant.



picture: energy audit

(a) Work against friction = 800 − 300 = 500 J

energy audit

(b) $10F = 500$

W = Fd

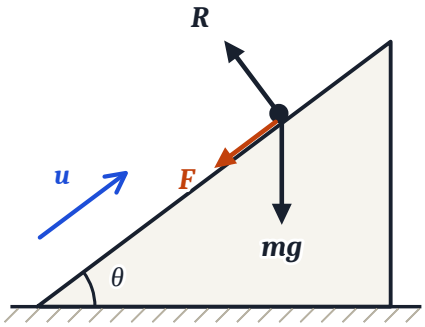
$F = 50$ N

÷ 10

D · Show-that, curved paths and time Use energy for speeds and distances; use Newton for time.

12 A particle is projected with speed u up a line of greatest slope of a rough plane inclined at θ to the horizontal. The coefficient of friction is μ.
(a) Show that it travels a distance $d = u^2/(2g(\sin\theta + \mu\cos\theta))$ before it stops.
(b) Find d when $u = 10$, $\sin\theta = 0.6$ and $\mu = 0.5$. Take $g = 9.8$ m s^{-2}.

(a) $R = mg\cos\theta$, so $F = \mu mg\cos\theta$ — R = mg cos θ, F = μR



picture: forces

$\frac{1}{2}mu^2 = mgd\sin\theta + \mu mg\cos\theta \times d$ — KE = GPE + work against

$\frac{1}{2}u^2 = gd(\sin\theta + \mu\cos\theta)$ — ÷ m

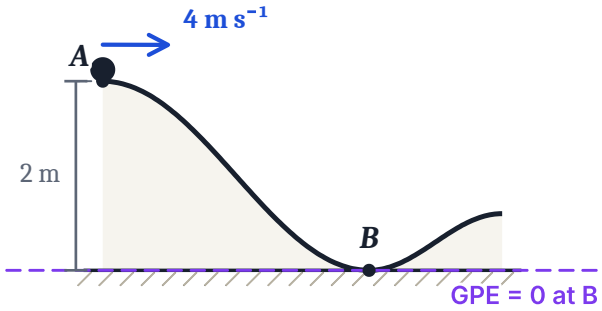
$d = u^2/(2g(\sin\theta + \mu\cos\theta))$ — ÷ g(sin θ + μ cos θ), × 2

(b) $\cos\theta = 0.8$ (3, 4, 5 triangle) — cos θ from sin θ

$d = 10^2/(2 \times 9.8 \times (0.6 + 0.5 \times 0.8)) = 5.10$ m

substitute, 3 s.f.

13 A particle of mass 0.5 kg moves on a rough curved track. It passes A at 4 m s^{-1}. Point B is 2 m below A. The work done against resistance from A to B is 3.2 J. Take $g = 9.8$ m s^{-2}. Find its speed at B.



picture: A is 2 m above B

GPE = 0 at B. Energy at A: $\frac{1}{2} \times 0.5 \times 4^2 + 0.5 \times 9.8 \times 2$
$= 4 + 9.8 = 13.8$ J — KE + GPE at A

KE at $B = 13.8 - 3.2 = 10.6$ J — energy audit

$\frac{1}{2} \times 0.5 \times v^2 = 10.6$, $v^2 = 42.4$ — KE = ½mv²

$v = 6.51$ m s^{-1} — square root, 3 s.f.

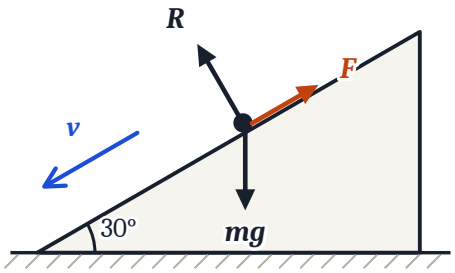
14 A particle is released from rest on a rough plane inclined at 30° to the horizontal. The coefficient of friction is 0.25. Take $g = 9.8$ m s^{-2}.
(a) Use energy to find its speed after it has slid 6 m.
(b) Use Newton's second law to find its acceleration, and hence the time taken.

(a) Per kilogram: GPE lost $= 9.8 \times 6 \sin 30° = 29.4$ J
÷ m: per kg

Work against friction $= 0.25 \times 9.8 \cos 30° \times 6 = 12.73...$ J
μg cos θ × d

$\frac{1}{2}v^2 = 29.4 - 12.73... = 16.66...$, $v^2 = 33.33...$
energy audit

$v = 5.77$ m s^{-1}
square root, 3 s.f.



picture: forces for F = ma

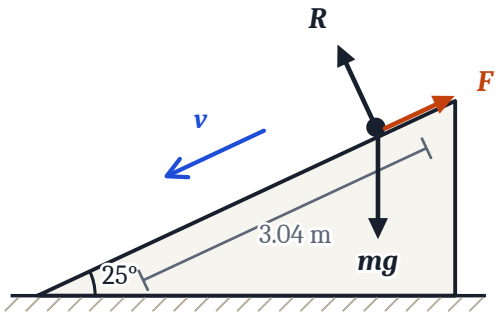
(b) $a = 9.8 \sin 30° - 0.25 \times 9.8 \cos 30° = 4.9 - 2.121...$
$= 2.778...$ m s^{-2}
F = ma, ÷ m

$t = 5.773... \div 2.778... = 2.08$ s
v = u + at

15 A particle is projected at 6 m s^{-1} up a line of greatest slope of a rough plane inclined at 25° to the horizontal. The coefficient of friction is 0.2. Take $g = 9.8$ m s^{-2}.
(a) Find the distance it travels before it stops.
(b) It slides back. Find its speed when it returns to its starting point.

(a) Per kilogram: $\frac{1}{2} \times 6^2 = s(9.8 \sin 25° + 0.2 \times 9.8 \cos 25°)$
KE = GPE + work against, ÷ m

$18 = s(4.141... + 1.776...) = 5.918...s$, $s = 3.04$ m
÷ 5.918...



picture: friction now acts up the slope

(b) Down 3.041... m: $\frac{1}{2}v^2 = 3.041... \times (4.141... - 1.776...)$
GPE lost − work against

$\frac{1}{2}v^2 = 3.041... \times 2.365... = 7.194...$, $v^2 = 14.38...$
× 2

$v = 3.79$ m s^{-1}
square root, 3 s.f.

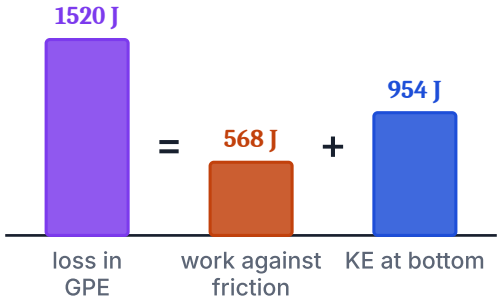
E · Exam-style question Show your method: marks are for working.

16 A sledge of mass 20 kg is released from rest at the top of a slope 30 m long, inclined at 15° to the horizontal. The coefficient of friction between the sledge and the slope is 0.1. At the bottom it moves, with no change in speed, on to rough horizontal ground with the same coefficient of friction. Take $g = 9.8$ m s^{-2}.
(a) Show that the work done against friction on the slope is 568 J, to 3 significant figures. [3]
(b) Use the work–energy principle to find the speed of the sledge at the bottom of the slope. [3]
(c) Find how far the sledge travels on the horizontal ground before it stops. [2]

(a) $R = 20 \times 9.8 \cos 15° = 189.3...$ N **M1**
R = mg cos θ

$F = 0.1 \times 189.3... = 18.93...$ N **M1**
F = μR

Work $= 18.93... \times 30 = 567.9... = 568$ J **A1**
W = Fd



picture: energy audit on the slope

(b) GPE lost $= 20 \times 9.8 \times 30 \sin 15° = 1521.8...$ J **M1**
h = d sin θ

$\frac{1}{2} \times 20 \times v^2 = 1521.8... - 567.9... = 953.8...$ **M1**
energy audit

$v = 9.77$ m s^{-1} **A1**
square root, 3 s.f.

(c) $F = 0.1 \times 20 \times 9.8 = 19.6$ N **M1**
R = mg on the level

$19.6d = 953.8...$, $d = 48.7$ m **A1**
work = KE lost