Power

Specification: AQA 7367: MC7 (with MC1–MC3) · Edexcel 9FM0: FM1 2.1 Work, energy and power

INDEPENDENT

You do

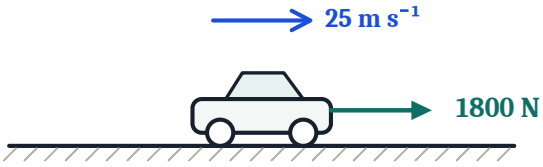
A · Power P = W/t; P = Fv; 1 kW = 1000 W.

1 A pump does 54 000 J of work in 1.5 minutes. Find its power.

1.5 minutes = 90 s — convert to seconds

$P = 54\,000 \div 90 = 600$ W — P = W/t

2 A car's engine produces a driving force of 1800 N when the car moves at 25 m s^{-1}. Find the power of the engine in kilowatts.

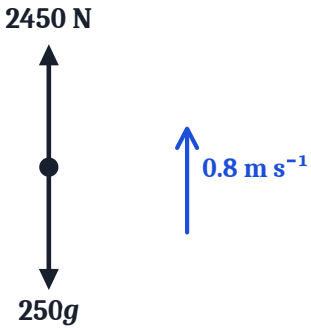


picture: driving force and speed

$P = Dv = 1800 \times 25 = 45\,000$ W — P = Fv

$P = 45$ kW — ÷ 1000

3 A crane lifts a load of mass 250 kg vertically at a constant speed of 0.8 m s^{-1}. Take $g = 9.8$ m s^{-2}. Find the power of the crane.



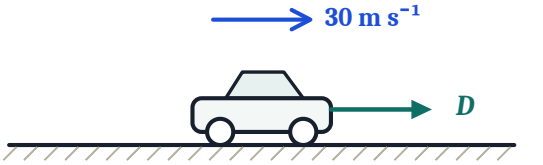
picture: T = mg at constant speed

Constant speed, so the tension = 250 × 9.8 = 2450 N — a = 0: T = mg

$P = 2450 \times 0.8 = 1960$ W — P = Fv

4 A train's engine works at 900 kW. Find the driving force when the train moves at 108 km h^{-1}.

108 ÷ 3.6 = 30 m s^{-1} — km h^{-1} ÷ 3.6

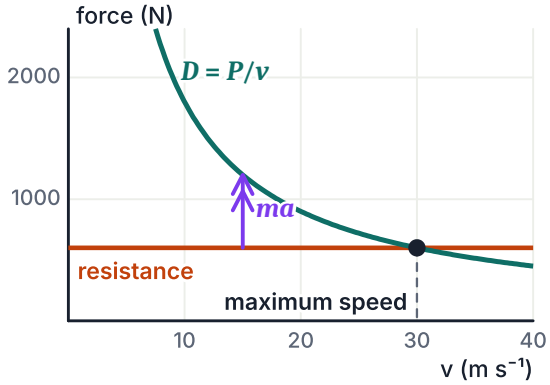


picture: D = P/v

$D = 900\,000 \div 30 = 30\,000$ N — D = P/v

B · Maximum speed and acceleration D = P/v; P/v − R (− mg sin θ) = ma; maximum speed when a = 0.

5 A car of mass 1000 kg moves on a level road. Its engine works at 18 kW and the resistance is a constant 600 N.
(a) Find the maximum speed.
(b) Find the acceleration when the speed is 15 m s^{-1}.



picture: D = P/v meets 600 N at 30 m s^{-1}

(a) $18\,000/v = 600$ — a = 0: P/v = R

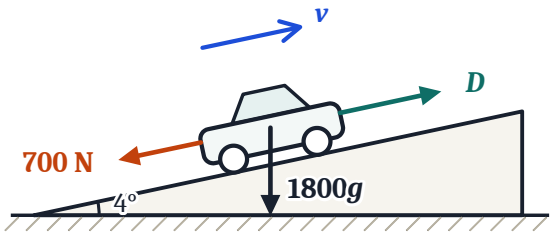
$v = 30$ m s^{-1} — ÷ 600

(b) $D = 18\,000 \div 15 = 1200$ N — D = P/v

$1000a = 1200 - 600 = 600$ — F = ma

$a = 0.6$ m s^{-2} — ÷ 1000

6 A van of mass 1800 kg climbs a road inclined at 4° to the horizontal. Its engine works at 26 kW against a constant resistance of 700 N. Take $g = 9.8$ m s^{-2}.
(a) Find the maximum speed up the road.
(b) Find the acceleration when the speed is 8 m s^{-1}.



picture: forces along the road

(a) Weight down the slope: $1800 \times 9.8 \sin 4° = 1230.5...$ N — mg sin θ

$D = 700 + 1230.5... = 1930.5...$ N — a = 0: D = R + mg sin θ

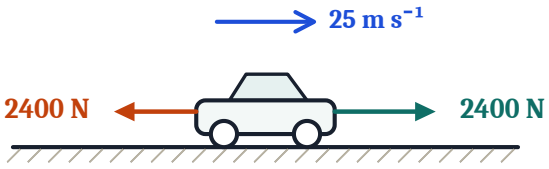
$v = 26\,000 \div 1930.5... = 13.5$ m s^{-1} — v = P/D, 3 s.f.

(b) $D = 26\,000 \div 8 = 3250$ N — D = P/v

$1800a = 3250 - 1930.5... = 1319.4...$ — F = ma up the slope

$a = 0.733$ m s^{-2} — ÷ 1800, 3 s.f.

7 A lorry of mass 6000 kg has a maximum speed of 25 m s^{-1} on a level road, against a constant resistance of 2400 N.
(a) Find the power of its engine.
(b) Working at this power, find its acceleration when its speed is 15 m s^{-1}.



picture: at maximum speed, D = R

(a) At maximum speed $D = 2400$ N — a = 0: D = R

$P = 2400 \times 25 = 60\,000$ W $= 60$ kW — P = Dv

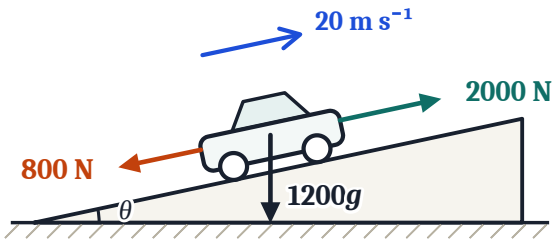
(b) $D = 60\,000 \div 15 = 4000$ N — D = P/v

$6000a = 4000 - 2400 = 1600$ — F = ma

$a = 0.267$ m s^{-2} — ÷ 6000, 3 s.f.

8 A car of mass 1200 kg has an engine of power 40 kW. The resistance is a constant 800 N. Its maximum speed up a hill inclined at θ to the horizontal is 20 m s^{-1}. Take $g = 9.8$ m s^{-2}. Find θ.

$D = 40\,000 \div 20 = 2000$ N — D = P/v



picture: forces along the road

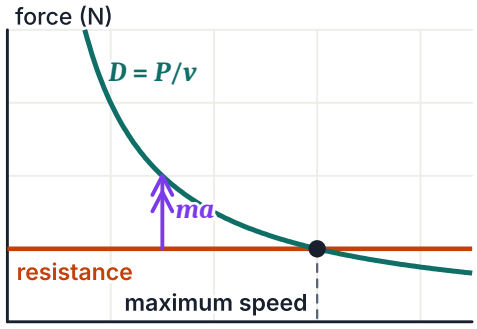
At maximum speed: $800 + 1200 \times 9.8 \sin\theta = 2000$ — a = 0: D = R + mg sin θ

$\sin\theta = 1200 \div 11\,760 = 0.1020...$ — − 800, ÷ 11 760

$\theta = 5.86°$ — inverse sin, 3 s.f.

C · Reasoning (no calculator) Explain in words as well as numbers.

9 A car's engine works at a constant power on a level road against a constant resistance. Explain why the car cannot keep accelerating.



picture: D falls until it meets R

The driving force is $D = P/v$: as the speed rises, D falls.

D = P/v falls

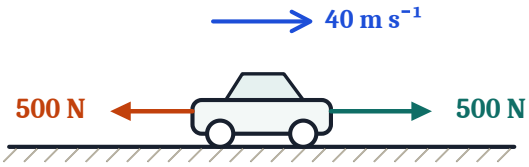
So the resultant force $D - R$ falls, and the acceleration falls.

F = ma

When $D = R$ the acceleration is zero: the car is at its maximum speed, P/R.

a = 0: top speed

10 A car of mass 1000 kg has a maximum speed of 40 m s^{-1} on a level road when its engine works at 20 kW. The resistance is constant.
(a) Find the resistance.
(b) Find the acceleration when the speed is 20 m s^{-1}.



picture: at maximum speed, D = R

(a) At maximum speed $a = 0$: $20\,000 = 40R$ — a = 0: P = Rv

$R = 500$ N — ÷ 40

(b) $D = 20\,000 \div 20 = 1000$ N — D = P/v

$1000a = 1000 - 500 = 500$ — F = ma

$a = 0.5$ m s^{-2} — ÷ 1000

11 A cyclist working at 200 W has a top speed of 10 m s^{-1} on level ground. The resistance is kv N at speed v m s^{-1}.
(a) Find k.
(b) Find the power needed for a top speed of 20 m s^{-1}. Comment on your answer.

(a) At top speed $D = kv$, so $P = kv \times v = kv^2$: $200 = 100k$

P = kv²

$k = 2$ — ÷ 100

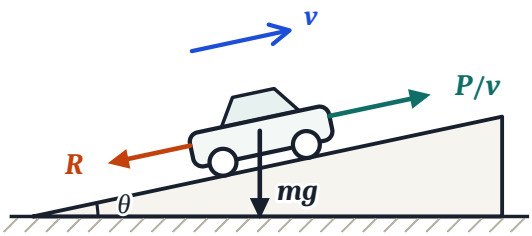
(b) $P = 2 \times 20^2 = 800$ W — P = kv²

Twice the speed needs four times the power.

comment

D · Show-that, kv and energy Set up F = ma along the road; use W = Pt for energy.

12 A vehicle of mass m climbs a road inclined at θ to the horizontal, working at a constant power P against a constant resistance R.
(a) Show that its maximum speed is $P/(R + mg\sin\theta)$.
(b) A car of mass 1200 kg has a maximum speed of 30 m s^{-1} on a level road and 20 m s^{-1} up a road with $\sin\theta = 1/20$, at the same power P and against the same resistance R. Take g = 9.8 m s^{-2}. Find R and P.



picture: forces along the road

(a) At maximum speed $a = 0$: $P/v - R - mg\sin\theta = 0$ — F = ma, a = 0

$P/v = R + mg\sin\theta$, so $v = P/(R + mg\sin\theta)$ — rearrange

(b) Level: $P = 30R$ — P = Rv

Slope: $mg\sin\theta = 1200 \times 9.8 \times 1/20 = 588$ N — mg sin θ

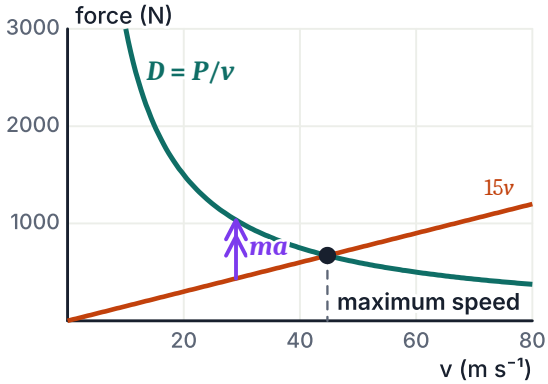
$P = 20(R + 588)$ — P = (R + mg sin θ)v

$30R = 20R + 11\,760$ — equate

$R = 1176$ N — ÷ 10

$P = 30 \times 1176 = 35\,280$ W — P = 30R

13 A car of mass 1200 kg works at 30 kW on a level road. The resistance is $15v$ N at speed v m s^{-1}.
(a) Find its maximum speed.
(b) Find its speed when its acceleration is 0.5 m s^{-2}.



picture: D = P/v meets 15v

(a) $30\,000/v = 15v$, so $v^2 = 2000$ — a = 0: P/v = kv

$v = 44.7$ m s^{-1} — square root, 3 s.f.

(b) $30\,000/v - 15v = 1200 \times 0.5 = 600$ — F = ma

$30\,000 - 15v^2 = 600v$ — × v

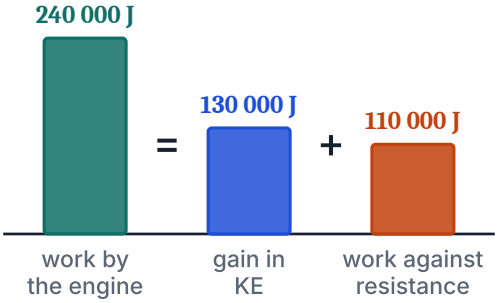
$v^2 + 40v - 2000 = 0$ — ÷ 15, rearrange

$v = -20 + \sqrt{2400} = 29.0$ m s^{-1} — positive root, 3 s.f.

14 A car of mass 1000 kg moves on a level road. Its engine works at a constant 20 kW for 12 s, and its speed rises from 8 m s^{-1} to 18 m s^{-1}.
(a) Find the work done by the engine.
(b) Find the work done against the resistance.
(c) Explain why you cannot use suvat to find the distance travelled.

(a) $W = Pt = 20\,000 \times 12 = 240\,000$ J

W = Pt



picture: energy audit

(b) KE gained = ½ × 1000 × $(18^2 - 8^2)$ = 130 000 J

KE gained

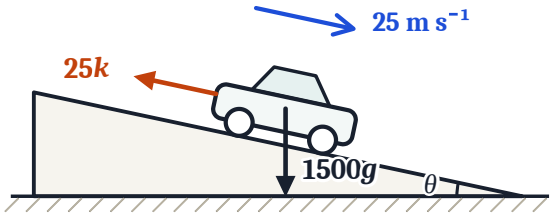
Work against resistance
= 240 000 − 130 000 = 110 000 J

energy audit

(c) The driving force P/v changes as v changes, so the acceleration is not constant.

a not constant

15 A car of mass 1500 kg has a resistance of kv N at speed v m s^{-1}. With the engine off, it rolls down a hill with $\sin\theta = 1/30$ at a constant 25 m s^{-1}. Take $g = 9.8$ m s^{-2}.
(a) Find k.
(b) Find the power needed to drive up the same hill at a constant 15 m s^{-1}.



picture: engine off, kv = mg sin θ

(a) Weight down the hill: 1500 × 9.8 × 1/30 = 490 N

mg sin θ

Constant speed, engine off: $25k = 490$

a = 0: kv = mg sin θ

$k = 19.6$

÷ 25

(b) Up the hill at 15 m s^{-1}: $D = 19.6 \times 15 + 490 = 784$ N

a = 0: D = kv + mg sin θ

$P = 784 \times 15 = 11\,760$ W

P = Dv

E · Exam-style question Show your method: marks are for working.

16 A lorry of mass 5000 kg has an engine with a maximum power of 80 kW. On a straight level road, with its engine at maximum power, its maximum speed is 32 m s^{-1}. The resistance to motion is a constant R N. Take $g = 9.8$ m s^{-2}.

(a) Show that $R = 2500$. [2]

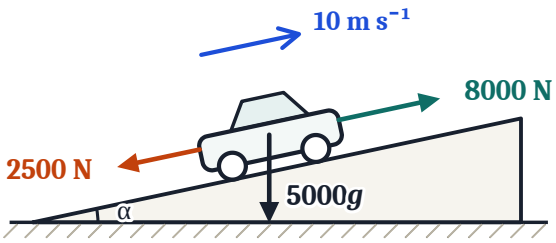
(b) The lorry climbs a straight hill inclined at α to the horizontal, where $\sin\alpha = 1/25$, with its engine at maximum power and the same resistance. Find its acceleration when its speed is 10 m s^{-1}. [3]

(c) Find the maximum speed of the lorry up the hill. [2]

(d) State one way in which the model for the resistance could be improved. [1]

(a) At maximum speed $D = R$, so $P = Rv$: $80\,000 = 32R$ **M1** — a = 0: P = Rv

$R = 2500$ N (AG) **A1** — ÷ 32



picture: forces up the hill

(b) $D = 80\,000 \div 10 = 8000$ N **B1** — D = P/v

Weight down the hill $= 5000 \times 9.8 \times 1/25 = 1960$ N — mg sin α

$5000a = 8000 - 2500 - 1960$ **M1** — F = ma up the hill

$5000a = 3540$, $a = 0.708$ m s^{-2} **A1** — ÷ 5000

(c) $80\,000/v = 2500 + 1960 = 4460$ **M1** — a = 0

$v = 17.9$ m s^{-1} **A1** — ÷ 4460, 3 s.f.

(d) The resistance would really increase with speed: model it as kv (or kv^2). **B1** — improve the model