Unit Review

Specification: AQA 7367: MC1, MC2, MC3, MC7 · Edexcel 9FM0: FM1 2.1 Work, energy and power

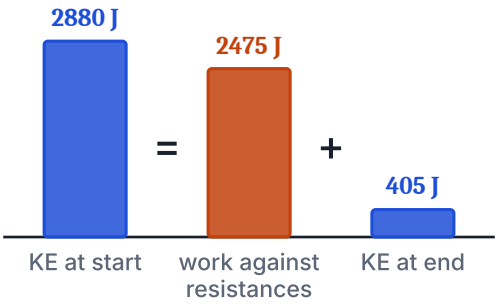
INDEPENDENT Mixed practice

A · Fluency One tool each. Which one?

1 A cyclist and her bicycle have a total mass of 90 kg. On a level road she stops pedalling and slows from 8 m s^{-1} to 3 m s^{-1}. Find the work done against the resistances.

KE at start = ½ × 90 × 8^2 = 2880 J (from L1) — KE = ½mv²

KE at end = ½ × 90 × 3^2 = 405 J — KE = ½mv²



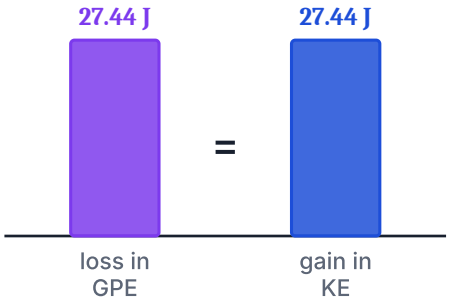
picture: KE at start = work against + KE at end

Work against resistances = 2880 − 405 = 2475 J

= KE lost

2 A ball of mass 0.4 kg is dropped from a height of 12 m. Ignore air resistance. Take g = 9.8 m s^{-2}. Use energy to find its speed when it has fallen 7 m.

GPE lost = 0.4 × 9.8 × 7 = 27.44 J (from L2) — GPE = mgh



picture: GPE lost = KE gained

½ × 0.4 × v^2 = 27.44 — KE gained = GPE lost

v^2 = 27.44 ÷ 0.2 = 137.2 — ÷ 0.2

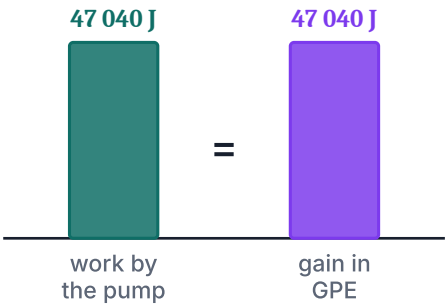
v = 11.7 m s^{-1} — square root, 3 s.f.

3 A pump raises 600 kg of water through a height of 8 m every minute. The water leaves the pump at a very low speed. Take g = 9.8 m s^{-2}. Find the power of the pump.

Work done each minute = GPE gained

= 600 × 9.8 × 8 = 47 040 J (from L4, L2)

GPE = mgh

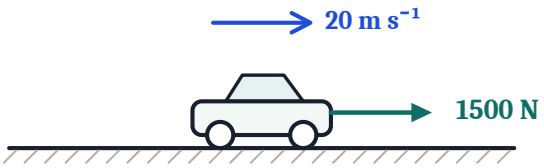


picture: work done = GPE gained

P = 47 040 ÷ 60 = 784 W — P = W/t, t in seconds

4 A car moves at 72 km h^{-1} on a level road. Its engine produces a driving force of 1500 N. Find the power of the engine in kilowatts.

72 ÷ 3.6 = 20 m s^{-1} (from L4) — km h^{-1} ÷ 3.6



picture: P = Dv

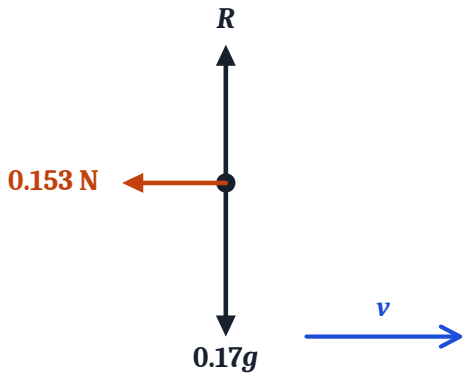
P = 1500 × 20 = 30 000 W — P = Fv

P = 30 kW — ÷ 1000

5 An ice-hockey puck of mass 0.17 kg slides 25 m in a straight line on horizontal ice. Its speed falls from 9 m s^{-1} to 6 m s^{-1}. Take $g = 9.8$ m s^{-2}.
(a) Find the work done against friction.
(b) Find the coefficient of friction between the puck and the ice.

(a) KE lost = ½ × 0.17 × ($9^2 - 6^2$) = 3.825 J (from L1)

work against = KE lost



picture: friction F = μR

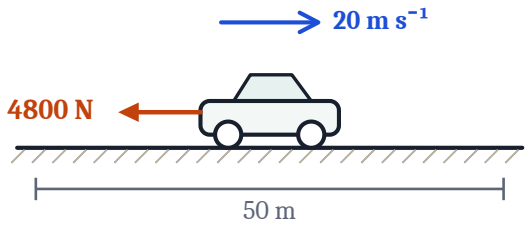
(b) $25F = 3.825$, $F = 0.153$ N — work = F × d

$R = 0.17 \times 9.8 = 1.666$ N — R = mg

μ = 0.153 ÷ 1.666 = 0.0918 — μ = F/R, 3 s.f.

6 A car of mass 1200 kg brakes on a level road. Its speed falls from 20 m s^{-1} to rest in 50 m. Use the work–energy principle to find the braking force, assumed constant.

KE lost = ½ × 1200 × 20^2 = 240 000 J (from L1) — KE = ½mv²



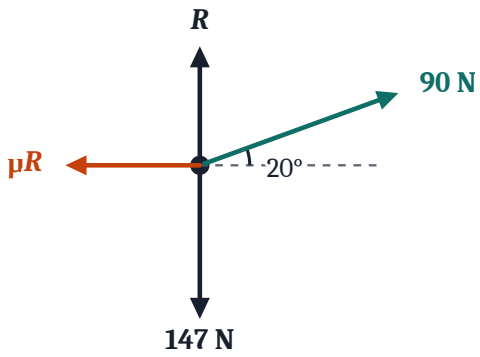
picture: braking force over 50 m

$50F = 240\,000$ — work against = KE lost

$F = 4800$ N — ÷ 50

B · Multi-step Name the tool; resolve where you need R; write the energy equation in words first.

7 A box of mass 15 kg is pulled from rest across a rough horizontal floor by a rope inclined at 20° above the horizontal. The tension in the rope is 90 N and the coefficient of friction is 0.4. Take $g = 9.8$ m s^{-2}. Find the speed of the box after it has moved 6 m.



picture: forces

$R = 15 \times 9.8 - 90 \sin 20° = 147 - 30.78... = 116.2...$ N (from L1) — resolve vertically

$F = 0.4 \times 116.2... = 46.48...$ N — F = μR

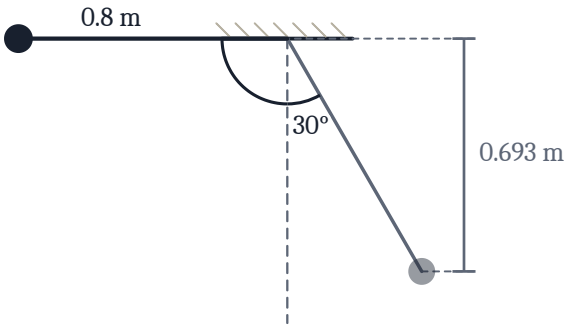
Work by the tension = 90 × 6 cos 20° = 507.4… J — W = Fd cos θ

Work against friction = 46.48… × 6 = 278.9… J — W = Fd

$½ \times 15 \times v^2 = 507.4... - 278.9... = 228.5...$ — work–energy principle

$v = 5.52$ m s^{-1} — ÷ 7.5, square root

8 A particle is attached to one end of a light inextensible string of length 0.8 m. The other end is fixed at O. The particle is held with the string taut and horizontal, and released from rest. Ignore air resistance. Take $g = 9.8$ m s^{-2}. Find the speed of the particle when the string makes an angle of 30° with the vertical.



picture: the drop is 0.8 cos 30°

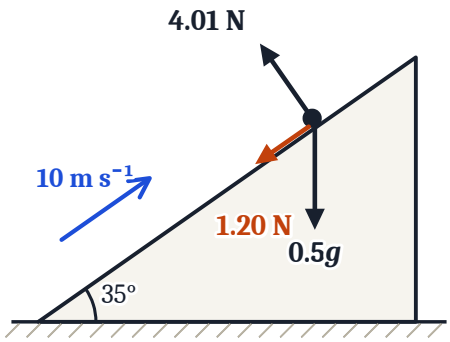
Drop = 0.8 cos 30° = 0.6928… m (from L2) — h = l cos θ

$½mv^2 = m \times 9.8 \times 0.6928...$ — KE gained = GPE lost

$v^2 = 2 \times 9.8 \times 0.6928... = 13.57...$ — ÷ m, × 2

$v = 3.69$ m s^{-1} — square root, 3 s.f.

9 A particle of mass 0.5 kg is projected with speed 10 m s^{-1} up a line of greatest slope of a rough plane inclined at 35° to the horizontal. The coefficient of friction is 0.3. Take $g = 9.8$ m s^{-2}.
(a) Find how far up the plane the particle travels.
(b) Find its speed when it returns to its starting point.



picture: friction acts down the slope

(a) $R = 0.5 \times 9.8 \cos 35° = 4.013...$ N, $F = 0.3R = 1.204...$ N
from L3

R = mg cos θ, F = μR

Weight down the slope $= 0.5 \times 9.8 \sin 35° = 2.810...$ N

mg sin θ

$\frac{1}{2} \times 0.5 \times 10^2 = (2.810... + 1.204...)d$, $25 = 4.014...d$

KE = GPE + work against

$d = 6.23$ m

÷ 4.014..., 3 s.f.

(b) $\mu = 0.3 < \tan 35° = 0.700...$: it slides back.

μ < tan θ: slides back

$\frac{1}{2} \times 0.5 \times v^2 = (2.810... - 1.204...) \times 6.227... = 10.00...$

GPE lost − work against

$v = 6.33$ m s^{-1}

÷ 0.25, square root

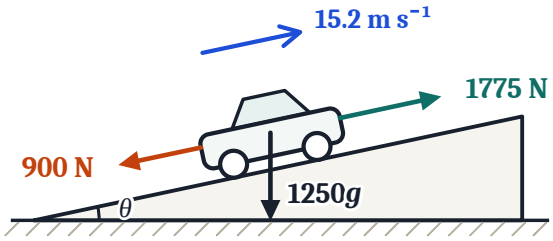
10 A car of mass 1250 kg has a maximum speed of 30 m s^{-1} on a level road when its engine works at 27 kW. The resistance to motion is constant. Take $g = 9.8$ m s^{-2}.
(a) Find the resistance.
(b) The car climbs a hill inclined at θ to the horizontal, where $\sin\theta = 1/14$, with its engine at 27 kW and the same resistance. Find its maximum speed up the hill.

(a) $a = 0$: $R = 27\,000 \div 30 = 900$ N
from L4

a = 0: R = P/v

(b) Weight down the hill $= 1250 \times 9.8 \times 1/14 = 875$ N

mg sin θ



picture: at maximum speed

$a = 0$: $D = 900 + 875 = 1775$ N

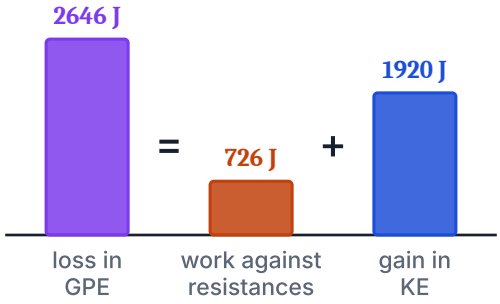
a = 0: D = R + mg sin θ

$v = 27\,000 \div 1775 = 15.2$ m s^{-1}

v = P/D, 3 s.f.

11 A skateboarder and her board have a total mass of 60 kg. She starts from rest at the top of a curved ramp 4.5 m above the ground and reaches the bottom at 8 m s^{-1}. Take $g = 9.8$ m s^{-2}.
(a) Find the work done against resistances on the ramp.
(b) She then moves along level ground against a constant resistance of 40 N. How far does she travel before she stops?

(a) GPE lost = $60 \times 9.8 \times 4.5 = 2646$ J (from L3, L1) — GPE = mgh
KE at the bottom = $\frac{1}{2} \times 60 \times 8^2 = 1920$ J — KE = ½mv²



picture: energy audit

Work against resistances = 2646 − 1920 = 726 J

energy audit

(b) $40d = 1920$ — work against = KE lost

$d = 48$ m — ÷ 40

C · Reasoning (no calculator) Explain in words as well as numbers.

12 Two identical balls are thrown from the top of the same cliff with the same speed. One is thrown horizontally and the other vertically upwards. Ignore air resistance. Explain why they hit the sea with the same speed.

Only the weight does work, so KE + GPE is constant for each ball. (from L2)

KE + GPE constant

They start with the same KE (same mass, same speed) and the same GPE.

same start

Both fall the same height to the sea, so both lose the same GPE and gain the same KE: the same speed.

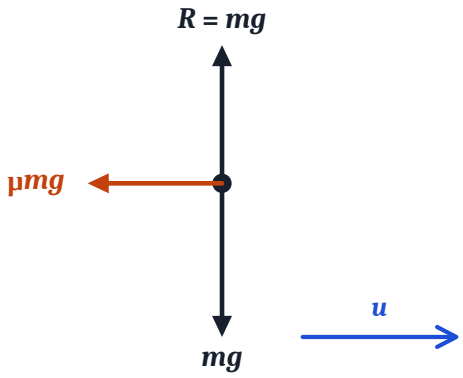
GPE lost = KE gained

Energy has no direction, so the direction of the throw does not matter.

energy: no direction

13 A box slides onto a rough horizontal floor with speed u and slides to rest. The coefficient of friction is μ.
(a) Show that the distance it slides does not depend on its mass.
(b) Find the distance when $u = 7$ m s^{-1} and $\mu = 0.25$. Take $g = 9.8$ m s^{-2}.

(a) $R = mg$, so $F = \mu mg$ (from L1) — R = mg, F = μR



picture: R = mg, F = μmg

Work against friction = KE lost: $\mu mg \times d = \frac{1}{2}mu^2$

work = KE lost

$d = u^2/(2\mu g)$: no m — ÷ m (mass cancels)

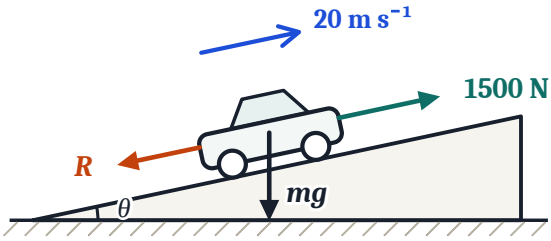
(b) $2\mu g = 2 \times 0.25 \times 9.8 = 4.9$ — 2μg

$d = 49 \div 4.9$, so $d = 10$ m — d = u²/(2μg)

14 A car climbs a hill at a constant speed of 20 m s^{-1} with its engine working at 30 kW.
(a) Find the driving force.
(b) Ali says: "The speed is constant, so the engine is doing no work." Explain why Ali is wrong.

(a) $D = P/v = 30\,000 \div 20 = 1500$ N from L4, L2

D = P/v



picture: forces along the road

(b) The engine does $30\,000$ J of work every second ($P = W/t$).

P = W/t

The KE stays the same, but the car gains GPE and does work against the resistance.

where the work goes

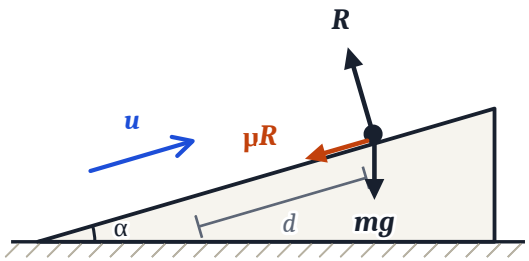
Work by the engine = GPE gained + work against resistance.

energy audit

D · Show that Start from the energy equation or F = ma; show every step.

15 A particle is projected with speed u up a line of greatest slope of a rough plane inclined at α to the horizontal. The coefficient of friction is μ.
(a) Show that it comes to rest after moving a distance $u^2/(2g(\sin\alpha + \mu\cos\alpha))$.
(b) Given $\tan\alpha = 7/24$, $\mu = 1/8$ and $u = 5.6$ m s^{-1}, find this distance. Take $g = 9.8$ m s^{-2}.
(c) Show that the particle slides back down, and find its speed when it returns to its starting point.

(a) $F = \mu R = \mu mg\cos\alpha$ from L3 — R = mg cos θ, F = μR



picture: friction acts down the slope

$\frac{1}{2}mu^2 = mgd\sin\alpha + \mu mg\cos\alpha \times d$ — KE = GPE + work against

$d = u^2/(2g(\sin\alpha + \mu\cos\alpha))$ — ÷ m, rearrange

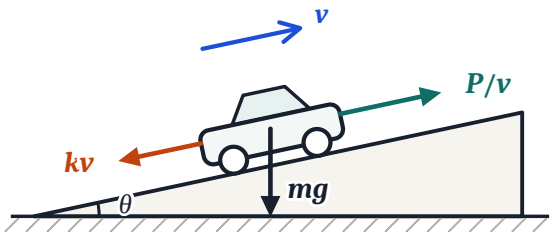
(b) $\sin\alpha = 7/25 = 0.28$, $\cos\alpha = 24/25 = 0.96$ — 7–24–25 triangle

$d = 5.6^2 \div (2 \times 9.8 \times (0.28 + 0.12)) = 4$ m — substitute

(c) $\mu = 0.125 < \tan\alpha = 0.29\ldots$: it slides back. — μ < tan θ: slides back

$v^2 = 2 \times 9.8 \times 4 \times (0.28 - 0.12) = 12.544$, $v = 3.54$ m s^{-1} — GPE lost − work against

16 A car of mass m works at a constant power P up a road inclined at θ to the horizontal. The resistance to motion is kv when its speed is v.
(a) Show that its maximum speed v satisfies $kv^2 + (mg\sin\theta)v - P = 0$.
(b) For a car with $m = 1000$ kg, $P = 36$ kW, $k = 24$ and $\sin\theta = 1/20$, find the maximum speed. Take $g = 9.8$ m s^{-2}.
(c) At this speed, find the rate at which the car gains gravitational potential energy.



picture: forces along the road

(a) At maximum speed $a = 0$: $P/v - kv - mg\sin\theta = 0$ from L4, L2 — F = ma, a = 0

× v: $kv^2 + (mg\sin\theta)v - P = 0$ — × v, rearrange

(b) $mg\sin\theta = 1000 \times 9.8 \times 1/20 = 490$ N — mg sin θ

$24v^2 + 490v - 36\,000 = 0$ — substitute

$v = (-490 + \sqrt{(490^2 + 4 \times 24 \times 36\,000)})/48$ — quadratic formula

$v = 29.8$ m s^{-1} — positive root, 3 s.f.

(c) Each second the car rises $v\sin\theta$, so the GPE rises at $mg\sin\theta \times v$ — rate of GPE = mg sin θ × v

$490 \times 29.84\ldots = 14\,623\ldots$ W $= 14.6$ kW — evaluate, 3 s.f.

E · Exam-style questions Show your method: marks are for working.

17 *AQA-style*. A winch pulls a crate of mass 200 kg up a line of greatest slope of a rough plane inclined at 15° to the horizontal. The cable is parallel to the slope. The coefficient of friction between the crate and the plane is 0.4. Take $g = 9.8$ m s^{-2}.

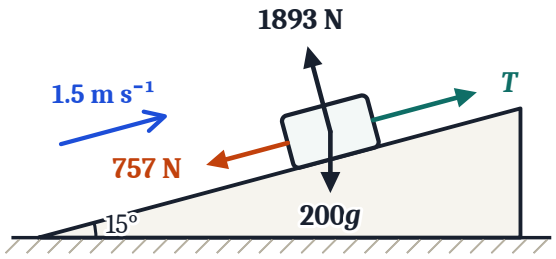
(a) Find the magnitude of the friction force on the crate. [2]

(b) The crate moves up the slope at a constant speed of 1.5 m s^{-1}. Find the power of the winch. [4]

(c) The crate is pulled 12 m up the slope at this speed. Find the work done by the winch and the gain in the crate's gravitational potential energy. [2]

(a) $R = 200 \times 9.8 \cos 15° = 1893.2...$ N from L3, L4, L2 **M1** — R = mg cos θ

$F = 0.4 \times 1893.2... = 757.2... = 757$ N (3 s.f.) **A1** — F = μR



picture: forces on the crate

(b) Weight down the slope $= 200 \times 9.8 \sin 15° = 507.2...$ N — mg sin θ

Constant speed: $T = 757.2... + 507.2...$ **M1** — a = 0: resolve along the slope

$T = 1264.5...$ N **A1** — add

$P = Tv = 1264.5... \times 1.5$ **M1** — P = Fv

$P = 1896.8...$ W $= 1.90$ kW **A1** — 3 s.f.

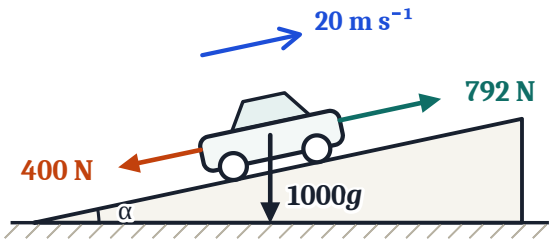
(c) Work by the winch $= 1264.5... \times 12 = 15\,174.8... = 15\,200$ J (3 s.f.) **B1** — W = Fd

GPE gained $= 200 \times 9.8 \times 12 \sin 15° = 6087.4... = 6090$ J (3 s.f.) **B1** — GPE = mgh, h = d sin θ

18 *Edexcel-style*. A car of mass 1000 kg moves up a straight road inclined at an angle α to the horizontal, where $\sin \alpha = 1/25$. The resistance to motion from non-gravitational forces is modelled as a constant force of magnitude 400 N. Take $g = 9.8$ m s^{-2}.

(a) The car moves up the road at a constant speed of 20 m s^{-1}. Find the rate at which the engine is working. [4]

(b) At the point A on the road the car's speed is 20 m s^{-1} and the engine is switched off. The car comes to rest at the point B. Use the work–energy principle to find the distance AB. [4]



picture: forces up the road

(a) Weight down the road $= 1000 \times 9.8 \times 1/25 = 392$ N from L4, L3 — mg sin α

Constant speed: $D = 400 + 392$ **M1** — a = 0: D = R + mg sin α

$D = 792$ N **A1** — add

$P = Dv = 792 \times 20$ **M1** — P = Dv

$P = 15\,840$ W $= 15.8$ kW **A1** — 3 s.f.

(b) KE at $A = \frac{1}{2} \times 1000 \times 20^2 = 200\,000$ J **B1** — KE = ½mv²

KE lost = GPE gained + work against resistance: $200\,000 = 392d + 400d$ **M1 A1** — energy audit, d = AB

$d = 200\,000 \div 792 = 252.5...$, $AB = 253$ m **A1** — ÷ 792, 3 s.f.