Velocity–Time Graphs

Specification: AQA 7357: Q1, Q2 · Edexcel 9MA0: 7.1, 7.2

INDEPENDENT You do

A · Acceleration from a gradient Gradient = change in velocity ÷ change in time.

1 A velocity–time graph is a straight line from $(0, 3)$ to $(5, 18)$, with v in m s^{-1} and t in seconds. Find the acceleration.

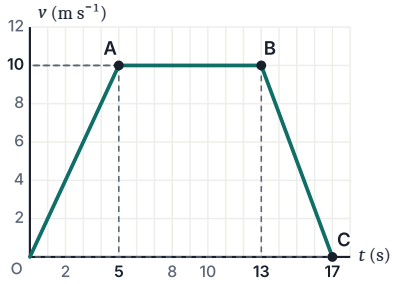
$(18 - 3) \div 5 = 3$ m s^{-2} — gradient = acceleration

2 A velocity–time graph is a straight line from $(2, 20)$ to $(10, 4)$ (v in m s^{-1}, t in s). Find the acceleration and the deceleration.

Acceleration $= (4 - 20) \div (10 - 2) = -2$ m s^{-2} — gradient = acceleration

Deceleration 2 m s^{-2} — slowing down

3 The graph shows the velocity of a robot. Find its acceleration on each stage OA, AB and BC.



OA: $10 \div 5 = 2$ m s^{-2} — gradient = acceleration

AB: 0 m s^{-2} — flat: constant velocity

BC: $-10 \div 4 = -2.5$ m s^{-2} — negative: deceleration

4 A car accelerates uniformly from 54 km h^{-1} to 90 km h^{-1} in 8 s. Find its acceleration in m s^{-2}.

$54 \div 3.6 = 15$, $90 \div 3.6 = 25$ m s^{-1} — km h^{-1} ÷ 3.6

$(25 - 15) \div 8 = 1.25$ m s^{-2} — change in v ÷ time

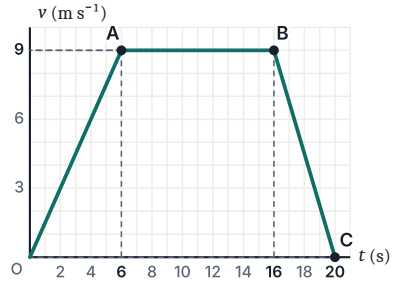
5 The velocity of a particle is $v = 12 - 3t$ (v in m s^{-1}, t in s). Find its acceleration, and when it is at rest.

Straight line, gradient -3: $a = -3$ m s^{-2} — gradient of v = 12 − 3t

$12 - 3t = 0 \Rightarrow t = 4$ s — at rest: v = 0

B · Displacement from the area Area under the graph = displacement.

6 The graph shows the velocity of a skier. Find the distance she travels.

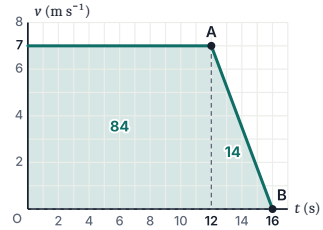


$\frac{1}{2} \times 6 \times 9 + 10 \times 9 + \frac{1}{2} \times 4 \times 9$ — triangle + rectangle + triangle

$= 27 + 90 + 18 = 135$ m — add the areas

7 A cyclist rides at a constant 7 m s^{-1} for 12 s, then decelerates uniformly to rest in 4 s.
(a) Sketch the velocity–time graph.
(b) Find the distance she travels.

(a)

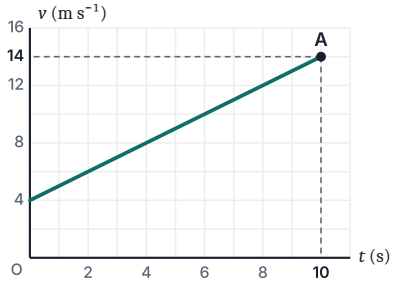


sketch

(b) $12 \times 7 + \frac{1}{2} \times 4 \times 7$ — rectangle + triangle

$= 84 + 14 = 98$ m — add the areas

8 The graph shows the velocity of a ball rolling down a slope.
(a) Find its acceleration.
(b) Find the distance it rolls in the 10 s.

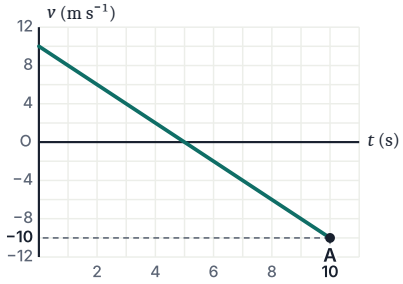


(a) $(14 - 4) \div 10 = 1$ m s^{-2} — gradient = acceleration

(b) Trapezium: $\frac{1}{2} \times (4 + 14) \times 10 = 90$ m — ½ × (sum of parallel sides) × width

C · Reasoning No calculator. Explain in words.

10 The graph shows the velocity of a particle.
(a) When does it turn round?
(b) Explain why it is back where it started at $t = 10$.
(c) Find the distance it travels.



(a) $t = 5$ s, where $v = 0$ (the graph crosses the t-axis) — v = 0: turns round

(b) The triangles above and below the axis are the same size, $\frac{1}{2} \times 5 \times 10 = 25$, so the displacement is $25 - 25 = 0$. — areas cancel

(c) $25 + 25 = 50$ m — distance: add sizes

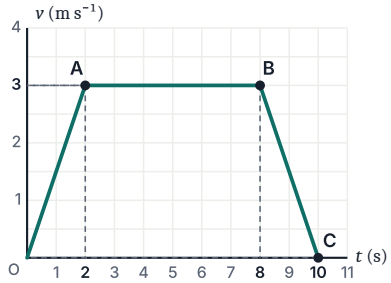
12 True or false? Explain each answer.
(a) If a velocity–time graph slopes down, the object is moving backwards.
(b) While an object moves in the positive direction, its velocity–time graph and its speed–time graph are the same.
(c) The total area between a velocity–time graph and the t-axis is always the displacement.

(a) False: it is slowing down (if $v > 0$). It moves backwards only when $v < 0$. — slope ≠ direction

(b) True: speed is the size of the velocity, and here $v \geq 0$. — speed = |v|

(c) False: area below the axis counts negative. Adding all the sizes gives the distance. — below counts −

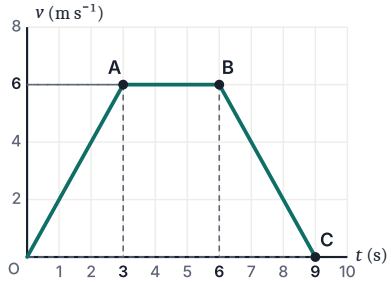
9 The graph shows the velocity of a lift moving up from the ground floor.
(a) Find the height the lift rises.
(b) Find its average velocity.



(a) $\frac{1}{2} \times (6 + 10) \times 3 = 24$ m — trapezium = displacement

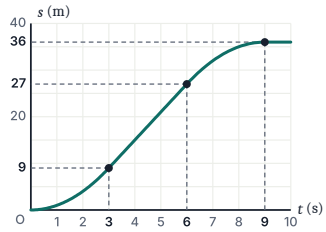
(b) $24 \div 10 = 2.4$ m s^{-1} — displacement ÷ time

11 Sketch the displacement–time graph for this velocity–time graph, marking the displacement at A, B and C. The object starts at O.



A: $\frac{1}{2} \times 3 \times 6 = 9$; B: $9 + 3 \times 6 = 27$; C: $27 + 9 = 36$ — area so far = s

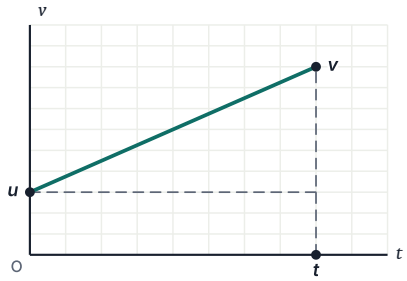
Curve getting steeper to A, straight from A to B, then less steep and flat at C. — v up, v steady, v down



sketch

D · Show that and unknowns Set up an equation from an area.

13 The velocity of an object increases uniformly from u to v in a time t. By splitting the area under its velocity–time graph into a rectangle and a triangle, show that its displacement is $\frac{1}{2}(u + v)t$.



Rectangle: $u \times t = ut$ — rectangle: height u

Triangle: $\frac{1}{2} \times t \times (v - u)$ — triangle: height v − u

Total = $ut + \frac{1}{2}vt - \frac{1}{2}ut = \frac{1}{2}ut + \frac{1}{2}vt = \frac{1}{2}(u + v)t$ — expand and collect

14 The velocity of a particle increases uniformly from 4 m s^{-1} to V m s^{-1} in 6 s. It travels 51 m in this time. Find V.

$\frac{1}{2} \times (4 + V) \times 6 = 51$ — area = displacement

$3(4 + V) = 51 \Rightarrow 4 + V = 17$ — simplify

$V = 13$ — solve

15 A car starts from rest and accelerates uniformly at 2.5 m s^{-2} until its speed is 20 m s^{-1}. It then keeps this speed. Show that it has travelled 200 m after 14 s.

Time to reach 20 m s^{-1}: $20 \div 2.5 = 8$ s — time = change in v ÷ a

Triangle: $\frac{1}{2} \times 8 \times 20 = 80$; rectangle: $(14 - 8) \times 20 = 120$ — area of each stage

$80 + 120 = 200$ m — add the areas

E · Exam-style question Show your method: marks are for working.

16 A particle P moves along a straight line from O. The graph shows its velocity v m s^{-1} after t seconds.

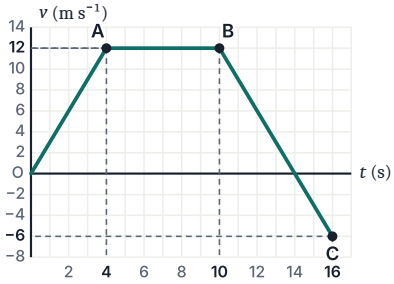
(a) Find the acceleration for the first 4 s. [1 mark]

(b) Find the deceleration on BC. [2 marks]

(c) Find when P is instantaneously at rest. [2 marks]

(d) Find the displacement from O at $t = 16$. [2 marks]

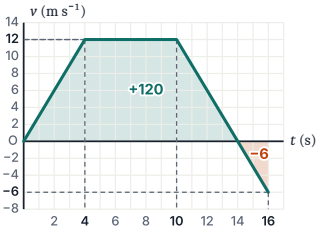
(e) Find the total distance travelled. [1 mark]



(a) $12 \div 4 = 3$ m s^{-2} **B1** — gradient = acceleration

(b) $(-6 - 12) \div 6 = -3$: deceleration 3 m s^{-2} **M1 A1** — gradient of BC

(c) $12 \div 3 = 4$ s after B: $t = 14$ s **M1 A1** — v = 0



areas: + above, − below

(d) $\frac{1}{2} \times (6 + 14) \times 12 - \frac{1}{2} \times 2 \times 6$ **M1** — trapezium − triangle

$= 120 - 6 = 114$ m **A1** — below counts −

(e) $120 + 6 = 126$ m **B1** — distance: add sizes