Multi-stage Velocity–Time Problems

Specification: AQA 7357: Q1, Q2 · Edexcel 9MA0: 7.1, 7.2

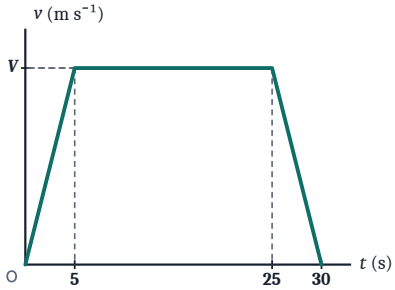
INDEPENDENT **You do**

A · An unknown time or speed Sketch, mark the times, then total area = total distance.

1 A car accelerates uniformly from rest to 10 m s^{-1} in 4 s. It travels at 10 m s^{-1} for T s, then decelerates uniformly to rest in 6 s. It travels 450 m. Find T.

Parallel sides T and $T + 10$ — parallel sides: the times
$\frac{1}{2}(T + T + 10) \times 10 = 10T + 50$ — trapezium in T
$10T + 50 = 450 \Rightarrow T = 40$ — area = distance

2 The graph shows the velocity of a car. It travels 375 m. Find V.



Parallel sides $25 - 5 = 20$ and 30 — parallel sides: the times
$\frac{1}{2}(20 + 30) \times V = 25V = 375$ — area = distance
$V = 15$ — solve

3 A particle accelerates uniformly from u m s^{-1} to 14 m s^{-1} in 5 s, then decelerates uniformly to rest in 7 s. It travels 109 m. Find u.

$\frac{1}{2}(u + 14) \times 5 + \frac{1}{2} \times 7 \times 14$ — trapezium + triangle
$= 2.5u + 35 + 49 = 2.5u + 84$ — simplify
$2.5u + 84 = 109 \Rightarrow u = 10$ — solve

4 A runner starts from rest and accelerates uniformly to V m s^{-1} in 4 s. She keeps this speed for 36 s, then decelerates uniformly to rest in 2 s. She runs 273 m. Find V.

Parallel sides 36 and $4 + 36 + 2 = 42$ — parallel sides: the times
$\frac{1}{2}(36 + 42) \times V = 39V = 273$ — area = distance
$V = 7$ — solve

5 A train starts from rest and accelerates uniformly to 30 m s^{-1} in 50 s. It travels at 30 m s^{-1} for T minutes, then decelerates uniformly to rest in 40 s. It travels 7.65 km. Find T.

7.65 km = 7650 m; T minutes = $60T$ s — units: m and s
$\frac{1}{2}(60T + 60T + 90) \times 30 = 1800T + 1350$ — trapezium in T
$1800T + 1350 = 7650 \Rightarrow T = 3.5$ — solve

B · Linked times Call the shorter time T. "4 times as long" is then 4T.

6 A car accelerates uniformly from rest to 8 m s^{-1} in 10 s. It travels at 8 m s^{-1} for 4 times as long as it then takes to decelerate uniformly to rest. It travels 400 m.
(a) Find the time it takes to decelerate.
(b) Find the total time.

(a) Decelerating T s; constant speed $4T$ s — shorter time = T
$40 + 32T + 4T = 40 + 36T = 400$ — add the areas
$T = 10$ s — solve

(b) $10 + 40 + 10 = 60$ s — add the times

7 A particle accelerates uniformly from rest to 12 m s^{-1} in T s. It moves at 12 m s^{-1} for $3T$ s, then decelerates uniformly to rest in $2T$ s. It travels 864 m. Find T and the total time.

Parallel sides $3T$ and $T + 3T + 2T = 6T$ — parallel sides: the times
$\frac{1}{2}(3T + 6T) \times 12 = 54T = 864$ — trapezium in T
$T = 16$; total time $6T = 96$ s — solve

8 A cyclist accelerates uniformly from rest for 6 s. She rides at a constant V m s^{-1} for 5 times as long, then decelerates uniformly to rest in 4 s. She travels 350 m.
(a) Find V.
(b) Find her acceleration.

(a) Constant speed for $5 \times 6 = 30$ s; parallel sides 30 and 40

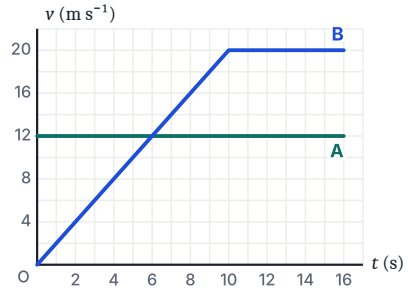
5 times as long

$\frac{1}{2}(30 + 40) \times V = 35V = 350 \Rightarrow V = 10$ area = distance

(b) $10 \div 6 = 1.67$ m s^{-2} gradient = acceleration

C · Two vehicles No calculator. Same start: level when the areas are equal.

9 Two cars A and B pass the same point O at $t = 0$. The graph shows their velocities.
(a) When do they have the same velocity?
(b) Explain why A is ahead at that time, and find by how much.
(c) Find when B draws level with A.



(a) B: $a = 20 \div 10 = 2$; $v = 12$ at $t = 6$ s graphs cross: same v

(b) A's area is bigger: $6 \times 12 = 72$ against $\frac{1}{2} \times 6 \times 12 = 36$. A is 36 m ahead.

compare the areas

(c) For $t > 10$: B $100 + 20(t - 10) = 20t - 100$; A $12t$

distance in t

$20t - 100 = 12t \Rightarrow t = 12.5$ s equal areas: level

10 Two vehicles start from the same point. True or false? Explain each answer.
(a) Where their velocity–time graphs cross, the vehicles are level.
(b) While the vehicle behind is moving faster, the gap gets smaller.
(c) They are level again when the areas under their velocity–time graphs are equal.

(a) False: they have the same velocity there, not the same position.

crossing: same v

(b) True: it covers more distance each second than the one in front.

faster one gains

(c) True: area = displacement, and they started at the same point.

equal areas: level

11 Two runners start together. P runs at a constant 6 m s^{-1}. Q starts from rest, accelerates uniformly to 8 m s^{-1} in 8 s and then keeps this speed.
(a) When is P farthest ahead, and by how much?
(b) When does Q catch P?

(a) Q: $a = 1$; same speed 6 at $t = 6$. Gap $\frac{1}{2} \times 6 \times 6 = 18$ m

same v: gap greatest

(b) At $t = 8$: P 48, Q 32. For $t > 8$: Q $32 + 8(t - 8) = 8t - 32$

distance in t

$8t - 32 = 6t \Rightarrow t = 16$ s equal areas: level

D · Show that and harder unknowns Write each time and speed in terms of the unknown first.

12 A car starts from rest and accelerates uniformly at 2 m s^{-2} for T s. It travels at constant speed for 20 s, then decelerates uniformly at 4 m s^{-2} to rest. It travels 550 m.
(a) Show that $3T^2 + 80T - 1100 = 0$.
(b) Find T.

(a) Top speed $2T$; slowing down takes $2T \div 4 = \frac{1}{2}T$ — speed and time in T
Parallel sides 20 and $T + 20 + \frac{1}{2}T = 1.5T + 20$ — parallel sides: the times
$\frac{1}{2}(1.5T + 40) \times 2T = 1.5T^2 + 40T = 550$ — area = distance
$\times 2$: $3T^2 + 80T - 1100 = 0$ — tidy up

(b) $(3T + 110)(T - 10) = 0 \Rightarrow T = 10$ $(T > 0)$ — factorise; T > 0

13 A particle starts from rest and accelerates uniformly for 5 s to V m s^{-1}. It then decelerates uniformly for 3 s to 4 m s^{-1}. It travels 54 m.
(a) Find V.
(b) Find its deceleration.

(a) $\frac{1}{2} \times 5 \times V + \frac{1}{2}(V + 4) \times 3$ — triangle + trapezium
$= 2.5V + 1.5V + 6 = 4V + 6 = 54 \Rightarrow V = 12$ — area = distance

(b) $(12 - 4) \div 3 = 2.67$ m s^{-2} — gradient, 3 s.f.

14 Car A passes a point O at 8 m s^{-1} and accelerates uniformly at 0.5 m s^{-2} until its speed is 14 m s^{-1}; it then keeps this speed. At the same moment car B passes O at a constant 12 m s^{-1}.
(a) Show that A is behind B when A reaches 14 m s^{-1}.
(b) Show that A draws level with B 18 s after passing O, and find how far from O.

(a) Time $6 \div 0.5 = 12$ s; A $\frac{1}{2}(8 + 14) \times 12 = 132$; B $12 \times 12 = 144$
A is 12 m behind. — area of each; compare

(b) For $t > 12$: A $132 + 14(t - 12) = 14t - 36$; B $12t$ — distance in t
$14t - 36 = 12t \Rightarrow t = 18$; $12 \times 18 = 216$ m — equal areas: level

15 A train journey takes 100 s. The train accelerates uniformly from rest to 20 m s^{-1}, travels at 20 m s^{-1}, then decelerates uniformly to rest. Slowing down takes twice as long as speeding up. It travels 1700 m.
(a) Find the time it spends speeding up.
(b) Find its acceleration and its deceleration.

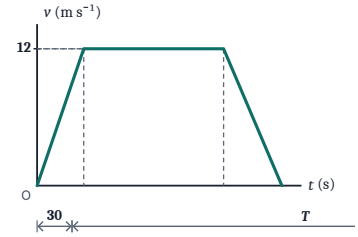
(a) Speeding up T, slowing down $2T$, constant speed $100 - 3T$
$\frac{1}{2}(100 - 3T + 100) \times 20 = 2000 - 30T = 1700$ — times in T; trapezium in T
$T = 10$ s — solve

(b) $20 \div 10 = 2$ m s^{-2}; deceleration $20 \div 20 = 1$ m s^{-2} — gradient = acceleration

E · Exam-style question Show your method: marks are for working.

16 A train starts from rest at station X and accelerates uniformly at 0.4 m s^{-2} for 30 s. It then travels at the velocity it has reached for T seconds, and then decelerates uniformly to rest in 45 s, stopping at station Y. The distance XY is 5.25 km.
(a) Sketch a velocity–time graph for the journey. [3 marks]
(b) Find T. [3 marks]
(c) Find the distance the train travels at constant velocity. [2 marks]

(a) Top speed $0.4 \times 30 = 12$ m s^{-1} **B1** — v = at



shape; 12; times marked
B1 B1

(b) $\frac{1}{2}(T + T + 75) \times 12$ (5.25 km = 5250 m) **M1** — trapezium in T
$12T + 450 = 5250$ **M1** — area = distance
$T = 400$ **A1** — solve

(c) 12×400 **M1** — rectangle
$= 4800$ m **A1** — with units