Quadratics in t, Turning Round, Two Particles

Specification: AQA 7357: Q1, Q3, B3 · Edexcel 9MA0: 7.1, 7.3, 2.3

INDEPENDENT

You do

A · Quadratics in t Substitute, collect on one side, solve: keep both positive roots.

1 A particle passes O with velocity 18 m s^{-1} and constant acceleration −4 m s^{-2}. Find the times when it is 36 m from O, on the positive side.

$36 = 18t - 2t^2 \Rightarrow t^2 - 9t + 18 = 0$ — s = ut + ½at², ÷ 2

$(t - 3)(t - 6) = 0 \Rightarrow t = 3$ s or $t = 6$ s

Do Now Q1: both times

2 A particle passes O with velocity 7 m s^{-1} and constant acceleration −2 m s^{-2}. When does it return to O?

$0 = 7t - t^2 = t(7 - t)$ — s = 0: back at O

$t = 7$ s ($t = 0$ is the start) — t = 0 is the start

3 A particle passes O with velocity 20 m s^{-1} and constant acceleration −4 m s^{-2}. The point A is 42 m from O in its direction of motion.
(a) Find the times when it is at A.
(b) For how long is it beyond A?

(a) $42 = 20t - 2t^2 \Rightarrow t^2 - 10t + 21 = 0$ — s = ut + ½at², ÷ 2

$(t - 3)(t - 7) = 0 \Rightarrow t = 3$ s or $t = 7$ s — factorise

(b) $7 - 3 = 4$ s — $t_2 - t_1$

4 A particle passes O with velocity 5 m s^{-1} and constant acceleration 2 m s^{-2}. Find the time it takes to travel 24 m.

$24 = 5t + t^2 \Rightarrow t^2 + 5t - 24 = 0$ — s = ut + ½at²

$(t + 8)(t - 3) = 0 \Rightarrow t = 3$ s ($t = -8$ rejected) — reject t < 0

5 A particle passes O with velocity 12 m s^{-1} and constant acceleration −3 m s^{-2}. Find, to 3 s.f., the times when it is 15 m from O, on the positive side.

$15 = 12t - 1.5t^2 \Rightarrow t^2 - 8t + 10 = 0$ — s = ut + ½at², ÷ 1.5

$t = \dfrac{8 \pm \sqrt{(64 - 40)}}{2} = 4 \pm \sqrt{6}$ — quadratic formula

$t = 1.55$ s or $t = 6.45$ s — calculator, 3 s.f.

B · Turning round and distance Find where v = 0; distance = out + back.

6 A particle passes O with velocity 14 m s^{-1} and constant acceleration −3.5 m s^{-2}.
(a) Find the time when it returns to O.
(b) Find the total distance it travels in this time.

(a) $0 = 14t - 1.75t^2 = t(14 - 1.75t) \Rightarrow t = 8$ s — s = 0; t = 0 is the start

(b) $0 = 14^2 - 7s \Rightarrow s = 28$ m — v = 0: furthest point

Distance $2 \times 28 = 56$ m — out and back

7 A particle passes O moving in the positive direction with speed 6 m s^{-1}. Its acceleration is a constant 1.5 m s^{-2} in the negative direction.
(a) Find, to 3 s.f., the time when it is 9 m from O on the negative side.
(b) Find the total distance it has travelled by then.

(a) $-9 = 6t - 0.75t^2 \Rightarrow t^2 - 8t - 12 = 0$ — s = −9; ÷ 0.75

$t = 4 + 2\sqrt{7} = 9.29$ s ($4 - 2\sqrt{7} < 0$ rejected) — quadratic formula; t > 0

(b) Turns at $t = 4$, 12 m out: $12 + 12 + 9 = 33$ m — out 12, back 12 + 9

8 A particle passes O with velocity 11 m s^{-1} and constant acceleration −2 m s^{-2}. The point A is 24 m from O in its direction of motion.
(a) Find the times when it is at A.
(b) Find its velocity at each of these times.
(c) Find its greatest distance from O on the positive side.

(a) $t^2 - 11t + 24 = 0 \Rightarrow (t-3)(t-8) = 0$: $t = 3, 8$ — s = ut + ½at²; factorise

(b) $v = 11 - 6 = 5$ m s^{-1}; $v = 11 - 16 = -5$ m s^{-1} — v = u + at

(c) $t = 5.5$: $s = 60.5 - 30.25 = 30.25$ m — v = 0 halfway between

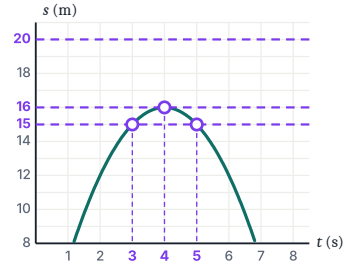
C · Reasoning No calculator.

10 A particle passes O with velocity 8 m s^{-1} and constant acceleration −2 m s^{-2}. Use $b^2 - 4ac$ to decide how many times it is at the point on the positive side
(a) 15 m from O
(b) 16 m from O
(c) 20 m from O.

(a) $t^2 - 8t + 15 = 0$: $64 - 60 > 0$: twice — b² − 4ac > 0: twice

(b) $t^2 - 8t + 16 = 0$: $64 - 64 = 0$: once (it turns there) — b² − 4ac = 0: once

(c) $t^2 - 8t + 20 = 0$: $64 - 80 < 0$: never — b² − 4ac < 0: never



check: 2, 1, 0 crossings

12 A particle Q passes a point A three seconds after a particle P does, moving in the same direction with speed 5 m s^{-1} and acceleration 2 m s^{-2}. Time t is measured from when P passes A.
(a) Write down the displacement of Q from A at time t.
(b) For which values of t is your expression valid?
(c) Find the displacement of Q from A when $t = 5$.

(a) $s_Q = 5(t-3) + (t-3)^2$ — t − 3 in place of t

(b) $t \geq 3$: before then Q has not reached A. — Q starts at t = 3

(c) $5 \times 2 + 2^2 = 14$ m — substitute t = 5

9 A particle passes O with velocity 15 m s^{-1} and constant acceleration −2.5 m s^{-2}. For how long is it more than 30 m from O on the positive side? Give your answer to 3 s.f.

$30 = 15t - 1.25t^2 \Rightarrow t^2 - 12t + 24 = 0$ — s = ut + ½at², ÷ 1.25

$t = 6 \pm 2\sqrt{3}$ — quadratic formula

$4\sqrt{3} = 6.93$ s — $t_2 - t_1$: exact roots

11 True or false? Explain each answer.
(a) If $s = ut + \frac{1}{2}at^2$ gives two positive values of t, one of them must be wrong.
(b) A particle with constant acceleration that passes a point twice turns round exactly halfway between the two times.
(c) A particle with constant acceleration, $u > 0$ and $a < 0$, returns to its start after twice the time it takes to stop.

(a) False: it passes the point going out, then again coming back. — both roots are real

(b) True: the roots are $-\frac{u}{a} \pm k$, and $v = 0$ at $t = -\frac{u}{a}$. — the graph is symmetrical

(c) True: $s = 0$ gives $t = -\frac{2u}{a}$; $v = 0$ gives $t = -\frac{u}{a}$. — s = 0 and v = 0

D · Two particles and "show that"

Same point, same direction, same clock.

13 Two particles P and Q move along the same straight line. At $t = 0$, P passes a point A with speed 2 m s^{-1} and acceleration 1 m s^{-2}. Three seconds later Q passes A in the same direction with speed 4 m s^{-1} and acceleration 4 m s^{-2}.

(a) Show that Q catches P when $3t^2 - 20t + 12 = 0$.

(b) Find the time when Q catches P. Explain why you reject one root.

(c) Find how far from A they are.

(a) $s_P = 2t + 0.5t^2$, $s_Q = 4(t - 3) + 2(t - 3)^2$ — Q: t − 3 in place of t

$2t + 0.5t^2 = 2t^2 - 8t + 6 \Rightarrow 3t^2 - 20t + 12 = 0$ — displacements equal; × 2

(b) $(3t - 2)(t - 6) = 0$: $t = 6$ s. $t = \frac{2}{3}$ is rejected: Q has not reached A ($t < 3$). — factorise; t ≥ 3

(c) $2 \times 6 + 0.5 \times 36 = 30$ m — distance from A

14 The points A and B are 30 m apart. At $t = 0$ a particle P passes A moving towards B with speed 6 m s^{-1} and acceleration 3 m s^{-2} towards B. At the same instant a particle Q passes B moving towards A with speed 5 m s^{-1} and acceleration 1 m s^{-2} towards A. Find when and where they meet.

$s_P = 6t + 1.5t^2$, $s_Q = 30 - 5t - 0.5t^2$ — A to B positive; both from A

$2t^2 + 11t - 30 = 0 \Rightarrow (2t + 15)(t - 2) = 0$ — displacements equal; factorise

$t = 2$ s; $12 + 6 = 18$ m from A — t > 0; distance from A

15 A particle passes O with velocity u m s^{-1}, where $u > 0$. Its acceleration is a constant $-k$ m s^{-2}, where $k > 0$.

(a) Show that it returns to O after $\frac{2u}{k}$ seconds.

(b) Show that by then it has travelled $\frac{u^2}{k}$ metres.

(a) $0 = ut - \frac{1}{2}kt^2 = t(u - \frac{1}{2}kt)$ — s = 0: back at O

$t = \frac{2u}{k}$ ($t = 0$ is the start) — t = 0 is the start

(b) $0 = u^2 - 2ks \Rightarrow s = \frac{u^2}{2k}$ — v = 0: furthest point

Distance $2 \times \frac{u^2}{2k} = \frac{u^2}{k}$ — out and back

E · Exam-style question

Show your method: marks are for working.

16 A particle P moves along a straight line with constant deceleration 2 m s^{-2}. At time $t = 0$ it passes a point O with speed 16 m s^{-1}, moving towards a point A, where OA = 55 m.

(a) Find the times when P is at A. [3 marks]

(b) Find the total distance P travels between these times. [3 marks]

(c) Find the speed of P when it returns to O. [2 marks]

(a) $55 = 16t - t^2$ M1 — s = ut + ½at², a = −2

$t^2 - 16t + 55 = 0 \Rightarrow (t - 5)(t - 11) = 0$ M1 — one side; factorise

$t = 5$ s and $t = 11$ s A1 — both times

(b) $0 = 16^2 - 4s \Rightarrow s = 64$ m M1 A1 — v = 0: furthest point

Out $64 - 55 = 9$, back 9: 18 m A1 — out and back

(c) $0 = 16t - t^2 \Rightarrow t = 16$; $v = 16 - 32 = -16$: speed 16 m s^{-1} M1 A1 — s = 0; v = u + at