Vertical Motion Under Gravity 1

Specification: AQA 7357: R3, Q3, OT3 · Edexcel 9MA0: 8.3, 7.3, OT3

INDEPENDENT

You do

A · Dropped and thrown down

Down positive: a = +9.8.

1 A stone is dropped from rest from the top of a building 40 m high. Find the time it takes to reach the ground.

$40 = 4.9t^2 \Rightarrow t^2 = 8.16...$ — s = ut + ½at², u = 0

$t = 2.85... = 2.9$ s — √, 2 s.f.

2 A coin is dropped from rest from a bridge and hits the water 1.5 s later. Find
(a) the height of the bridge above the water
(b) the speed of the coin when it hits the water.

(a) $s = 4.9 \times 1.5^2 = 11.025$, 11 m — s = ut + ½at², 2 s.f.

(b) $v = 9.8 \times 1.5 = 14.7$ m s^{-1} — v = u + at

3 A ball is thrown vertically downwards with speed 6 m s^{-1} from a window 15 m above the ground. Find the speed with which it hits the ground.

$v^2 = 6^2 + 2 \times 9.8 \times 15 = 330$ — v² = u² + 2as

$v = 18.1... = 18$ m s^{-1} — √, 2 s.f.

4 A stone is thrown vertically downwards with speed 2 m s^{-1} from the top of a cliff. It hits the sea 3 s later. Find the height of the cliff.

$s = 2 \times 3 + 4.9 \times 3^2 = 6 + 44.1 = 50.1$ — s = ut + ½at²

The cliff is 50 m high. — 2 s.f.

B · Thrown up

Up positive: a = −9.8 all the way. At the top v = 0.

5 A ball is projected vertically upwards with speed 7 m s^{-1}. Find
(a) the greatest height above the point of projection
(b) the time taken to reach it.

(a) $0 = 7^2 - 19.6s \Rightarrow s = 2.5$ m — v = 0; v² = u² + 2as

(b) $0 = 7 - 9.8t \Rightarrow t = 0.714... = 0.71$ s — v = u + at, 2 s.f.

6 A ball is projected vertically upwards from ground level with speed 17.5 m s^{-1}. Find
(a) the greatest height
(b) the time of flight.

(a) $0 = 17.5^2 - 19.6s \Rightarrow s = 15.6... = 16$ m — v = 0; 2 s.f.

(b) $0 = 17.5t - 4.9t^2 = t(17.5 - 4.9t)$ — s = 0: back down

$t = 3.57... = 3.6$ s — t = 0 is the start

7 A ball is projected vertically upwards. Two seconds later it is moving downwards with speed 5.6 m s^{-1}. Find
(a) the speed of projection
(b) the greatest height above the point of projection.

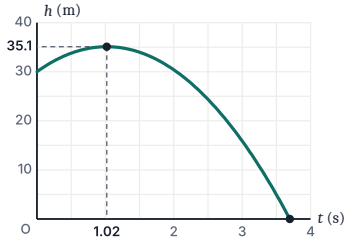
(a) $-5.6 = u - 9.8 \times 2 \Rightarrow u = 14$ m s^{-1} — v = −5.6: moving down

(b) $0 = 14^2 - 19.6s \Rightarrow s = 10$ m — v = 0 at the top

8 A stone is thrown vertically upwards with speed 10 m s^{-1} from the edge of a cliff 30 m above the sea. Find
(a) the time it takes to reach the sea
(b) its speed when it hits the sea.

(a) $-30 = 10t - 4.9t^2 \Rightarrow 4.9t^2 - 10t - 30 = 0$ — s = −30: lands below

$t = \dfrac{10 + \sqrt{(100 + 588)}}{9.8} = 3.69... = 3.7$ s — quadratic formula; t > 0



check: lands at 3.7 s

(b) $v^2 = 10^2 + 2(-9.8)(-30) = 688 \Rightarrow v = -26.2...$: 26 m s^{-1} — v² = u² + 2as; v < 0

9 A ball is thrown vertically upwards with speed 12 m s^{-1}.
(a) Find its speed when it is 5 m above the point of projection.
(b) Explain why the ball has this speed twice.

(a) $v^2 = 12^2 - 2 \times 9.8 \times 5 = 46 \Rightarrow v = \pm 6.78...$ $v^2 = u^2 + 2as$
Speed 6.8 m s^{-1} speed = |v|, 2 s.f.

(b) It passes that height going up ($v = +6.8$) and again coming down ($v = -6.8$).

± : up and down

C · Reasoning No calculator.

10 True or false? Explain each answer.
(a) At its greatest height, a ball thrown upwards has zero acceleration.
(b) A ball projected upwards from the ground at 9.8 m s^{-1} reaches its greatest height after 1 s and lands after 2 s.
(c) In the model, a heavy ball dropped from a tower lands before a light ball dropped at the same time.

(a) False: $v = 0$ for an instant, but gravity still acts: $a = -9.8$ (up positive).

a never changes

(b) True: $0 = 9.8 - 9.8t$ gives $t = 1$; $0 = 9.8t - 4.9t^2$ gives $t = 2$.

time up = time down

(c) False in the model: g does not depend on mass. (In real life air resistance can matter.)

g: the same for all masses

11 Take $g = 10$ m s^{-2}. A ball is thrown vertically upwards with speed 20 m s^{-1} from a point 25 m above the ground. Find
(a) the greatest height above the ground
(b) the time of flight
(c) the speed with which it hits the ground.

(a) $0 = 400 - 20s \Rightarrow s = 20$; $20 + 25 = 45$ m v = 0; add 25 m

(b) $-25 = 20t - 5t^2 \Rightarrow t^2 - 4t - 5 = 0$ s = −25; ÷ (−5)
$(t - 5)(t + 1) = 0 \Rightarrow t = 5$ s factorise; t > 0

(c) $v = 20 - 10 \times 5 = -30$: 30 m s^{-1} v = u + at

12 A tennis ball and a flat sheet of paper are dropped at the same time from the same height.
(a) What does the model of this lesson predict?
(b) What actually happens? Which modelling assumption fails?
(c) Why are answers found with $g = 9.8$ given to 2 s.f.?

(a) They land together: same $u = 0$, same $a = 9.8$, same distance.

g: the same for all masses

(b) The ball lands first. Air resistance on the paper is large compared with its weight: "no air resistance" fails.

air resistance

(c) $g = 9.8$ is only given to 2 s.f. (it varies with place), so more figures would claim an accuracy the model does not have.

accuracy matches g

D · Algebra and "show that" Letters: u, h, g. Up positive, so a = −g.

13 A ball is projected vertically upwards from the ground with speed u m s^{-1}. Show that

(a) it reaches its greatest height, $\frac{u^2}{2g}$ metres, after $\frac{u}{g}$ seconds

(b) it lands after $\frac{2u}{g}$ seconds.

(a) $0 = u^2 - 2gs \Rightarrow s = \frac{u^2}{2g}$ — v = 0; a = −g

$0 = u - gt \Rightarrow t = \frac{u}{g}$ — v = u + at

(b) $0 = ut - \frac{1}{2}gt^2 = t(u - \frac{1}{2}gt) \Rightarrow t = \frac{2u}{g}$ — s = 0; t = 0 is the start

14 A stone is thrown vertically upwards with speed 4.9 m s^{-1} from a point 9.8 m above the ground. Show that its time of flight, T seconds, satisfies $T^2 - T - 2 = 0$, and find T.

$-9.8 = 4.9T - 4.9T^2$ — s = −9.8: lands below

$-2 = T - T^2 \Rightarrow T^2 - T - 2 = 0$ — ÷ 4.9; one side

$(T - 2)(T + 1) = 0 \Rightarrow T = 2$ — factorise; T > 0

15 A particle is projected vertically with speed u m s^{-1} from a point h metres above the ground.
(a) Show that it hits the ground with speed $\sqrt{(u^2 + 2gh)}$, whether it is projected upwards or downwards.
(b) Find this speed when $u = 7$ and $h = 20$.

(a) Up positive: $s = -h$, $a = -g$, initial velocity u or $-u$. — up positive

$v^2 = (\pm u)^2 + 2(-g)(-h) = u^2 + 2gh$ — v² = u² + 2as: u² either way

Speed $\sqrt{(u^2 + 2gh)}$ both times. — speed = |v|

(b) $\sqrt{(49 + 392)} = \sqrt{441} = 21$ m s^{-1} — substitute

E · Exam-style question Show your method: marks are for working.

16 A ball is projected vertically upwards with speed 15.4 m s^{-1} from a point A, which is 8 m above the ground. The ball is modelled as a particle moving freely under gravity.
(a) Find the greatest height above the ground reached by the ball. [3 marks]
(b) Find the time from projection until the ball hits the ground. [3 marks]
(c) Find the speed with which the ball hits the ground. [2 marks]

(a) $0 = 15.4^2 - 19.6s \Rightarrow s = 12.1$ M1 A1 — v = 0; v² = u² + 2as

$12.1 + 8 = 20.1$, 20 m A1 — add the 8 m; 2 s.f.

(b) $-8 = 15.4t - 4.9t^2$ M1 — s = −8: lands below

$4.9t^2 - 15.4t - 8 = 0 \Rightarrow t = \frac{15.4 + \sqrt{393.96}}{9.8}$ M1 — quadratic formula

$t = 3.59... = 3.6$ s A1 — reject t < 0; 2 s.f.

(c) $v^2 = 15.4^2 + 2(-9.8)(-8) = 393.96 \Rightarrow v = -19.8...$, 20 m s^{-1} M1 A1 — v² = u² + 2as; v < 0