Vertical Motion Under Gravity 2

Specification: AQA 7357: R3, Q2, Q3, OT3 · Edexcel 9MA0: 8.3, 7.2, 7.3, OT3

INDEPENDENT

You do

A · Time above a height Solve $s = ut - 4.9t^2$ for the height: two roots. Time above = $t_2 - t_1$.

1 A ball is projected vertically upwards from the ground with speed 14.7 m s^{-1}.
(a) Find the two times at which it is 9.8 m above the ground.
(b) For how long is it above 9.8 m?

(a) $9.8 = 14.7t - 4.9t^2 \Rightarrow t^2 - 3t + 2 = 0$ ÷ 4.9
$(t-1)(t-2) = 0$: $t = 1$ or $t = 2$ factorise

(b) $2 - 1 = 1$ s $t_2 - t_1$

2 A ball is projected vertically upwards from a point O with speed 21 m s^{-1}. Find the total time for which it is at least 18 m above O.

$4.9t^2 - 21t + 18 = 0$ s = 18; one side
$t = \dfrac{21 \pm \sqrt{88.2}}{9.8} = 1.18\ldots$ or $3.10\ldots$ quadratic formula
$3.10\ldots - 1.18\ldots = 1.91\ldots, 1.9$ s $t_2 - t_1$; 2 s.f.

3 A stone is projected vertically upwards from a point O with speed 15 m s^{-1}. Find the length of time for which it is more than 5 m above O.

$4.9t^2 - 15t + 5 = 0$ s = 5; one side
$t = \dfrac{15 \pm \sqrt{127}}{9.8} = 0.380\ldots$ or $2.68\ldots$ quadratic formula
$2.68\ldots - 0.380\ldots = 2.29\ldots, 2.3$ s $t_2 - t_1$; 2 s.f.

4 A ball is projected vertically upwards from a point O with speed 12 m s^{-1}.
(a) Show that it never reaches a point 8 m above O.
(b) Find its greatest height above O.

(a) $4.9t^2 - 12t + 8 = 0$: $b^2 - 4ac = 144 - 156.8 < 0$
$b^2 - 4ac < 0$: no roots
No real t, so it never gets to 8 m. never reaches

(b) $0 = 12^2 - 19.6s \Rightarrow s = 7.34\ldots, 7.3$ m v = 0 at the top

B · Two objects; bounces One origin, one positive direction, t from the first start. Each bounce is a new motion.

5 Ball A is dropped from rest from the top of a tower 24 m high. At the same instant ball B is projected vertically upwards with speed 12 m s^{-1} from the foot of the tower, directly below A. Find when the balls meet and their height above the ground then.

$h_A = 24 - 4.9t^2$, $h_B = 12t - 4.9t^2$ origin: the ground
$12t = 24 \Rightarrow t = 2$ s equal heights; t^2 cancels
$h_B = 24 - 19.6 = 4.4$ m substitute

6 A stone is dropped from rest from the top of a cliff. Two seconds later a second stone is thrown vertically downwards from the same point with speed 29.4 m s^{-1}.
(a) Assuming the cliff is high enough, find when the second stone would catch the first, measured from when the first is dropped.
(b) How far below the top would they be then?
(c) The cliff is 70 m high. Do the stones meet before they reach the sea?

Down positive: $s_1 = 4.9t^2$, $s_2 = 29.4(t-2) + 4.9(t-2)^2$ same clock: t − 2

(a) $0 = 29.4t - 58.8 - 19.6t + 19.6 \Rightarrow 9.8t = 39.2$, $t = 4$ s expand; t^2 cancels

(b) $s_1 = 4.9 \times 4^2 = 78.4$ m substitute

(c) No: 78.4 > 70. The first stone reaches the sea at $\sqrt{(70 \div 4.9)} = 3.77\ldots$ s, before $t = 4$.
check: still in the air?

7 Particle P is projected vertically upwards from a point O with speed 19.6 m s^{-1}. Two seconds later particle Q is projected vertically upwards from O with the same speed. Find the time, measured from when P is projected, at which they meet, and their height above O then.

$s_P = 19.6t - 4.9t^2$, $s_Q = 19.6(t-2) - 4.9(t-2)^2$ same clock: t − 2
$0 = -39.2 + 19.6t - 19.6 \Rightarrow 19.6t = 58.8$, $t = 3$ s expand; t^2 cancels
$s_P = 58.8 - 44.1 = 14.7$ m substitute

8 Ball A is projected vertically upwards from the ground with speed 20 m s^{-1}. At the same instant ball B is dropped from rest from a point 30 m directly above A.
(a) Find when the balls meet.
(b) Find the height above the ground at which they meet.
(c) Find the velocity of each ball when they meet.

(a) $20t - 4.9t^2 = 30 - 4.9t^2 \Rightarrow t = 1.5$ s equal heights

(b) $30 - 4.9 \times 1.5^2 = 18.975, 19$ m substitute; 2 s.f.

(c) $v_A = 20 - 14.7 = 5.3$ (up), $v_B = -14.7$ (down)
v = u + at; signs

9 A ball is dropped from rest from a height of 6 m. It rebounds with 0.7 of the speed with which it hits the ground. Find
(a) the speed with which it first hits the ground
(b) the greatest height after the first bounce
(c) the time from release until it hits the ground for the second time.

(a) $v^2 = 2 \times 9.8 \times 6 = 117.6 \Rightarrow v = 10.8\ldots$, 11 m s^{-1} $v^2 = u^2 + 2as$

(b) Rebound $0.7 \times 10.8\ldots = 7.59\ldots$; $s = 7.59\ldots^2 \div 19.6 = 2.94$, 2.9 m

new motion: v = 0 at the top

(c) $10.8\ldots \div 9.8 + 2 \times 7.59\ldots \div 9.8 = 1.10\ldots + 1.54\ldots = 2.65\ldots$, 2.7 s

down, then up and down

C · Reasoning No calculator.

10 Take $g = 10$ m s^{-2}. A ball is dropped from rest from a height of 20 m. Each time it hits the ground it rebounds with half the speed with which it hits the ground.
(a) Find the speed with which it first hits the ground.
(b) Find the greatest height after the first bounce.
(c) Explain why each greatest height is a quarter of the one before.

(a) $v^2 = 2 \times 10 \times 20 = 400 \Rightarrow 20$ m s^{-1} $v^2 = u^2 + 2as$

(b) Rebound 10 m s^{-1}; $0 = 100 - 20s \Rightarrow s = 5$ m new motion

(c) It lands with the speed it left the ground with, so each rebound speed is half the one before. Height $= v^2 \div 2g$, so each height is ¼ of the one before.

same height, same speed; $\propto v^2$

11 (a) A ball is thrown vertically upwards and caught at the same height. What is the gradient of its velocity–time graph (up positive)?
(b) Why is the area above the t-axis equal to the area below?
(c) A pupil sketches the velocity–time graph of a bouncing ball as one straight line. What is wrong?

(a) -9.8: $a = -9.8$ going up and coming down. gradient = a

(b) The areas are the height risen and the height fallen; it ends where it started, so the displacement (total area) is 0.

area = displacement

(c) At each bounce the velocity jumps from $-v$ up to the rebound velocity. The graph is a new line after each bounce (parallel, gradient -9.8).

a bounce: a jump

12 (a) Two balls are thrown vertically upwards from the same point at the same instant with different speeds. Explain why they cannot meet again while both are in the air.
(b) Two balls are thrown vertically upwards from the same point with speed 14.7 m s^{-1}, the second one second after the first. Use symmetry to find when they meet and how high they are.

(a) $h_A - h_B = (u_A - u_B)t$, which is 0 only at $t = 0$. t^2 cancels

(b) The first is at the top at $t = 1.5$. They meet at the same height, the first coming down, the second going up: $t - 1.5 = 1.5 - (t - 1)$.

symmetry about the top

$t = 2$ s; $h = 14.7 \times 2 - 4.9 \times 4 = 9.8$ m substitute

D · Algebra and "show that" Letters: u, h, H, e, g. Up positive, so a = −g.

13 A ball is projected vertically upwards from O with speed u m s^{-1}.

(a) Show that the total time for which it is at least h metres above O is $\frac{2}{g}\sqrt{(u^2 - 2gh)}$.

(b) Check your formula with $u = 24.5$, $h = 19.6$.

(a) $h = ut - \frac{1}{2}gt^2 \Rightarrow \frac{1}{2}gt^2 - ut + h = 0$ — s = ut + ½at², a = −g

$t = \dfrac{u \pm \sqrt{(u^2 - 2gh)}}{g}$ — quadratic formula

$t_2 - t_1 = \dfrac{2\sqrt{(u^2 - 2gh)}}{g}$ — subtract the roots

(b) $2\sqrt{(600.25 - 384.16)} \div 9.8 = 2 \times 14.7 \div 9.8 = 3$ s

as Worked example 1b

14 Ball A is dropped from rest from a height H metres. At the same instant ball B is projected vertically upwards with speed u m s^{-1} from the ground directly below A.

(a) Show that they meet after $\frac{H}{u}$ seconds.

(b) Show that they meet while B is still going up if $u^2 > gH$.

(a) $H - \frac{1}{2}gt^2 = ut - \frac{1}{2}gt^2 \Rightarrow t = \dfrac{H}{u}$ — equal heights; t² cancels

(b) B is going up until $t = \dfrac{u}{g}$. — v = 0 at the top

$\dfrac{H}{u} < \dfrac{u}{g} \Leftrightarrow gH < u^2$ — × ug (positive)

15 A ball is dropped from rest from a height of H metres. Each time it hits the ground it rebounds with e times the speed with which it hits the ground, where $0 < e < 1$. Show that its greatest height after the first bounce is e^2H, and after the second bounce e^4H.

$v^2 = 2gH$: impact speed $\sqrt{(2gH)}$ — v² = u² + 2as

Rebound $e\sqrt{(2gH)}$; height = $e^2 \times 2gH \div 2g = e^2H$

new motion: v = 0 at the top

The same step again from e^2H: $e^2 \times e^2H = e^4H$ — repeat

E · Exam-style question Show your method: marks are for working.

16 Ball P is projected vertically upwards with speed 19.6 m s^{-1} from a point A on the ground. One second later ball Q is projected vertically upwards from A with speed 24.5 m s^{-1}. The balls are modelled as particles moving freely under gravity.

(a) Find the total time for which P is at least 14.7 m above A. [3 marks]

(b) Find the time, measured from when P is projected, at which P and Q are at the same height, and find that height. [4 marks]

(c) State the velocity of P at this time. [1 mark]

(a) $14.7 = 19.6t - 4.9t^2 \Rightarrow t^2 - 4t + 3 = 0$ **M1** — s = 14.7; ÷ 4.9

$(t - 1)(t - 3) = 0$: $3 - 1 = 2$ s **M1 A1** — $t_2 - t_1$

(b) $s_P = 19.6t - 4.9t^2$, $s_Q = 24.5(t - 1) - 4.9(t - 1)^2$ **M1** — same clock: t − 1

$19.6t = 24.5t - 24.5 + 9.8t - 4.9 \Rightarrow 14.7t = 29.4$ **M1** — expand; t² cancels

$t = 2$ s; $h = 39.2 - 19.6 = 19.6$ m **A1 A1** — substitute

(c) $v = 19.6 - 9.8 \times 2 = 0$: P is at its greatest height. **B1** — v = u + at