Unit Review

Specification: AQA 7357: P1, Q1, Q2, Q3, R3, OT3 · Edexcel 9MA0: 6.1, 7.1, 7.2, 7.3, 8.3, OT3

INDEPENDENT Mixed practice

A · Fluency One step each. Which tool?

1 A car slows down uniformly from 72 km h^{-1} to 18 km h^{-1} in 6 s. Find its deceleration.

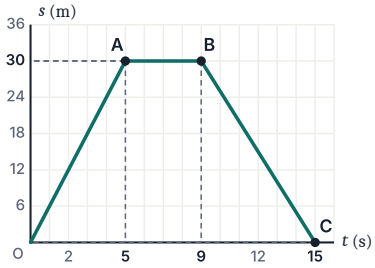
$72 \div 3.6 = 20$, $18 \div 3.6 = 5$ from L1, L4 — km h^{-1} ÷ 3.6

s	u	v	a	t
–	20	5	?	6

s u v a t

$5 = 20 + 6a \Rightarrow a = -2.5$: deceleration 2.5 m s^{-2} — v = u + at; a < 0

2 The graph shows the displacement of a cyclist from O.
(a) Find her velocity from B to C.
(b) Find her average speed for the whole journey.



(a) $(0 - 30) \div (15 - 9) = -5$ m s^{-1} from L1 — gradient = velocity

(b) Distance $30 + 30 = 60$ m — distance: out and back

$60 \div 15 = 4$ m s^{-1} — distance ÷ time

3 A stone is dropped from rest from a bridge and hits the water 1.5 s later. Find
(a) the height of the bridge above the water
(b) the speed with which the stone hits the water.

Down positive:

s	u	v	a	t
?	0	?	9.8	1.5

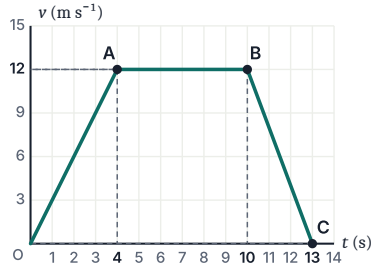
from L7

down positive: a = 9.8

(a) $s = \frac{1}{2} \times 9.8 \times 1.5^2 = 11.025$, 11 m — s = ut + ½at²; 2 s.f.

(b) $v = 9.8 \times 1.5 = 14.7$ m s^{-1} — v = u + at

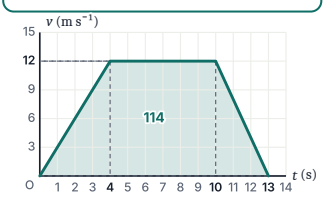
4 The graph shows the velocity of a particle.
(a) Find its acceleration from O to A.
(b) Find its deceleration from B to C.
(c) Find the total distance travelled.



(a) $12 \div 4 = 3$ m s^{-2} from L2 — gradient = a

(b) $12 \div 3 = 4$ m s^{-2} — size of the gradient

(c) $\frac{1}{2} \times (6 + 13) \times 12 = 114$ m — trapezium area



area = distance

5 A particle moving with constant acceleration 1.5 m s^{-2} passes a point P with speed 4 m s^{-1}. Find its distance from P when its speed is 10 m s^{-1}.

s	u	v	a	t
?	4	10	1.5	–

from L5 — s u v a t: no t

$10^2 = 4^2 + 2 \times 1.5 \times s \Rightarrow 100 = 16 + 3s$ — $v^2 = u^2 + 2as$

$s = 28$ m — solve

6 A particle moves with constant acceleration from A to B, a distance of 54 m, in 6 s. Its speed at B is 12 m s^{-1}. Find
(a) its speed at A
(b) its acceleration.

s	u	v	a	t
54	?	12	?	6

from L4 — s u v a t

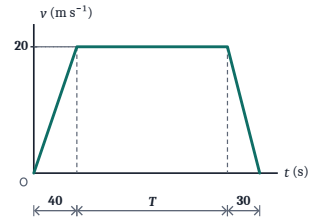
(a) $54 = \frac{1}{2}(u + 12) \times 6 \Rightarrow 18 = u + 12 \Rightarrow u = 6$ m s^{-1} — s = ½(u + v)t

(b) $12 = 6 + 6a \Rightarrow a = 1$ m s^{-2} — v = u + at

B · Multi-step Sketch first. Several tools in one question.

7 A train starts from rest at station A. It accelerates uniformly for 40 s to a speed of 20 m s^{-1}, travels at this speed for T seconds, then decelerates uniformly for 30 s, coming to rest at station B. The distance AB is 3.5 km.
(a) Sketch a velocity–time graph of the journey.
(b) Find T.
(c) Find the deceleration.

(a) Three stages: up to 20, level, down to 0 from L3 — sketch from words

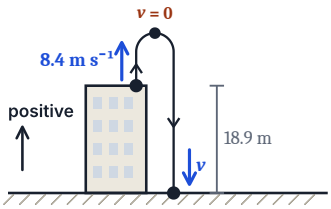


mark the times

(b) $\frac{1}{2} \times 40 \times 20 + 20T + \frac{1}{2} \times 30 \times 20 = 3500$ — total area = 3500 m
$400 + 20T + 300 = 3500 \Rightarrow T = 140$ — solve

(c) $20 \div 30 = 0.666\ldots$, 0.667 m s^{-2} — size of the gradient

8 A ball is projected vertically upwards with speed 8.4 m s^{-1} from a balcony 18.9 m above the ground. Find
(a) the greatest height of the ball above the ground
(b) the time it takes to reach the ground
(c) its speed as it hits the ground.



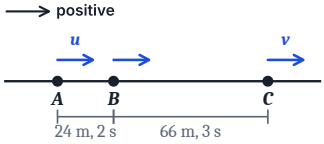
sketch; up positive

(a) $0 = 8.4^2 - 19.6s \Rightarrow s = 3.6$; $18.9 + 3.6 = 22.5$ m from L7 — v = 0 at the top

(b) $-18.9 = 8.4t - 4.9t^2 \Rightarrow 4.9t^2 - 8.4t - 18.9 = 0$ — s = −18.9; rearrange
$7t^2 - 12t - 27 = 0 \Rightarrow (7t + 9)(t - 3) = 0$, $t = 3$ s — ÷ 0.7; factorise; t > 0

(c) $v = 8.4 - 9.8 \times 3 = -21$: 21 m s^{-1} — v = u + at; speed

9 A particle moves with constant acceleration along a straight line through points A, B and C, in that order. $AB = 24$ m and $BC = 66$ m. It takes 2 s to go from A to B and 3 s to go from B to C.
(a) Find its acceleration and its speed at A.
(b) Find its speed at C.

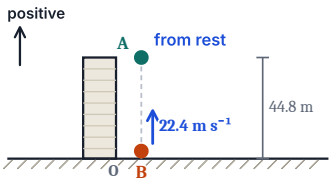


sketch; positive →

(a) A to B: $24 = 2u + 2a \Rightarrow u = 12 - a$ from L5 — s = ut + ½at²
A to C: $90 = 5u + 12.5a$ — A to C: 5 s
$90 = 60 - 5a + 12.5a \Rightarrow a = 4$ m s^{-2}, $u = 8$ m s^{-1} — substitute; solve

(b) $v = 8 + 4 \times 5 = 28$ m s^{-1} — v = u + at

10 Ball A is dropped from rest from the top of a tower 44.8 m high. At the same instant ball B is projected vertically upwards with speed 22.4 m s^{-1} from the foot of the tower, directly below A.
(a) Find when the balls meet.
(b) Find their height above the ground then.
(c) Is B moving up or down when they meet?



sketch: one origin

$h_A = 44.8 - 4.9t^2$, $h_B = 22.4t - 4.9t^2$ from L8 — h = h₀ + ut + ½at²

(a) $22.4t = 44.8 \Rightarrow t = 2$ s — equal heights; t² cancels

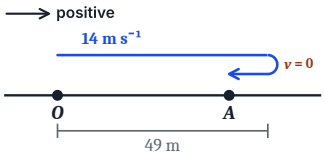
(b) $44.8 - 4.9 \times 2^2 = 25.2$ m — substitute

(c) $v_B = 22.4 - 19.6 = 2.8$: still going up. — v = u + at; sign

11 A particle P passes O with velocity 14 m s^{-1} in the positive direction. It has constant acceleration −2 m s^{-2}.
(a) Find the times at which P is 40 m from O on the positive side.
(b) Find the total distance travelled by P in the first 10 s.

(a) $40 = 14t - t^2 \Rightarrow t^2 - 14t + 40 = 0$ from L6 — s = ut + ½at²
$(t - 4)(t - 10) = 0$: $t = 4$ or $t = 10$ s — factorise: passes twice

(b) $0 = 14 - 2t \Rightarrow$ turns at $t = 7$, $s = 98 - 49 = 49$ — v = 0: turns round



out-and-back sketch

$49 + (49 - 40) = 58$ m — out + back

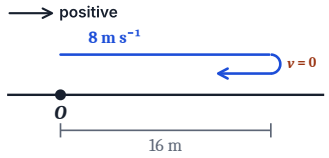
C · Reasoning No calculator.

12 A particle passes O with velocity 8 m s^{-1} and has constant acceleration −2 m s^{-2}.
(a) When does it turn round?
(b) Find its displacement from O after 6 s.
(c) Find the distance it travels in the 6 s, and explain why it is not the same as the displacement.

(a) $0 = 8 - 2t \Rightarrow t = 4$ s from L4, L6 — v = 0: turns round

(b) $s = 8 \times 6 - 36 = 12$ m — s = ut + ½at²

(c) At $t = 4$: $s = 32 - 16 = 16$. Out 16, back 4: 20 m — out + back



out-and-back sketch

Displacement is where it is; distance counts the 4 m back as well.

distance ≠ displacement

13 A ball is thrown vertically upwards with speed 9.8 m s^{-1}. Ali takes up as positive; Beth takes down as positive.
(a) Write down the acceleration each of them uses.
(b) Ali writes $a = -9.8$ on the way up and $a = +9.8$ on the way down. Explain the mistake.
(c) Using Beth's direction, find the ball's velocity after 3 s. Which way is it moving?

(a) Ali $a = -9.8$, Beth $a = +9.8$ m s^{-2} from L7 — a points down

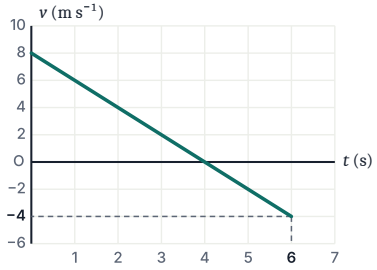
(b) Gravity pulls down all the time, so with up positive $a = -9.8$ going up, at the top and coming down.

sign of a never changes

(c) Beth: $u = -9.8$, $v = -9.8 + 9.8 \times 3 = 19.6$: 19.6 m s^{-1} downwards.

v = u + at; down +

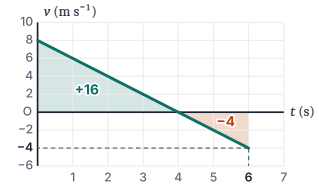
14 The graph shows the velocity of a particle that starts at O.
(a) When does it turn round?
(b) How far from O does it get?
(c) Find its displacement and the distance travelled after 6 s.
(d) At $t = 5$, is it speeding up or slowing down?



(a) $v = 0$ at $t = 4$ s from L2 — v crosses the t-axis

(b) ½ × 4 × 8 = 16 m — area above the axis

(c) 16 − ½ × 2 × 4 = 12 m; distance 16 + 4 = 20 m — + above, − below



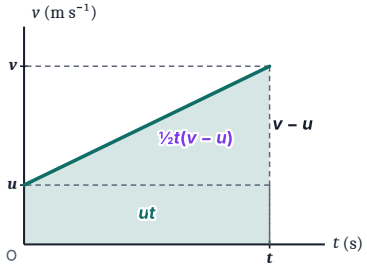
areas: + above, − below

(d) Speeding up: $v = -2$ and its size is growing, although $a < 0$.

speed = size of v

D · Derive and "show that" Work from the graph or from the formulae you know.

15 The graph shows a particle accelerating uniformly from u to v in time t.
(a) Use the graph to show that $s = ut + \frac{1}{2}at^2$.
(b) Using $v = u + at$ and $s = \frac{1}{2}(u + v)t$, show that $v^2 = u^2 + 2as$.



(a) s = area = rectangle + triangle = $ut + \frac{1}{2}t(v - u)$ from L4, L5
area = displacement
$v - u = at$ (the gradient is a) gradient = a
$s = ut + \frac{1}{2}t \times at = ut + \frac{1}{2}at^2$ substitute

(b) $t = (v - u) \div a$ into $s = \frac{1}{2}(u + v)t$ make t the subject
$s = (v + u)(v - u) \div 2a = (v^2 - u^2) \div 2a$ difference of two squares
$2as = v^2 - u^2 \Rightarrow v^2 = u^2 + 2as$ × 2a; rearrange

16 A cyclist starts from rest and accelerates uniformly at 0.5 m s^{-2} for T seconds. She then rides at the speed she has reached for 30 s. She travels 400 m altogether.
(a) Show that $T^2 + 60T - 1600 = 0$.
(b) Find T and her greatest speed.

(a) Stage 1: $s = \frac{1}{2} \times 0.5 \times T^2 = \frac{1}{4}T^2$, speed $0.5T$ from L4, L5
s = ut + ½at²; v = u + at
Stage 2: $0.5T \times 30 = 15T$ constant speed × time
$\frac{1}{4}T^2 + 15T = 400 \Rightarrow T^2 + 60T - 1600 = 0$ add the stages; × 4

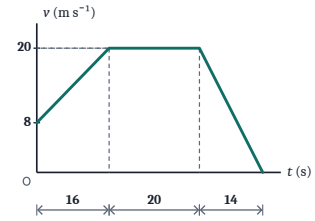
(b) $(T + 80)(T - 20) = 0 \Rightarrow T = 20$ factorise; T > 0
Greatest speed $0.5 \times 20 = 10$ m s^{-1} v = 0.5T

E · Exam-style questions Show your method: marks are for working.

17 *AQA-style*. A car passes a point A with speed 8 m s^{-1}. It accelerates uniformly at 0.75 m s^{-2} for 16 s, then travels at constant speed for 20 s. It then decelerates uniformly, coming to rest at a point B 14 s later.
(a) Show that the greatest speed of the car is 20 m s^{-1}. [2 marks]
(b) Sketch a velocity–time graph for the motion from A to B. [2 marks]
(c) Find the distance AB. [3 marks]
(d) Find the deceleration of the car. [1 mark]

(a) $v = 8 + 0.75 \times 16 = 20$ m s^{-1} from L2, L3, L4 **M1 A1** v = u + at

(b) Starts at 8, up to 20, level, down to 0 sketch from words



shape; values marked
B1 B1

(c) $\frac{1}{2}(8 + 20) \times 16 + 20 \times 20 + \frac{1}{2} \times 14 \times 20 = 224 + 400 + 140 = 764$ m **M1 A1 A1** area under the graph

(d) $20 \div 14 = 1.428\ldots$, 1.43 m s^{-2} **B1** size of the gradient

18 *Edexcel-style*. A ball is projected vertically upwards with speed 19.6 m s^{-1} from a point A which is 24.5 m above the ground. The ball is modelled as a particle moving freely under gravity. Find
(a) the greatest height of the ball above the ground, (2)
(b) the total time for which the ball is at least 39.2 m above the ground, (3)
(c) the time taken for the ball to reach the ground. (3)

(a) $0 = 19.6^2 - 19.6s \Rightarrow s = 19.6$; $24.5 + 19.6 = 44.1$ m from L7, L8 **M1 A1** v = 0 at the top; + 24.5

(b) $39.2 - 24.5 = 14.7$: $14.7 = 19.6t - 4.9t^2$ **M1** height above A
$t^2 - 4t + 3 = 0 \Rightarrow t = 1$ or 3 **M1** ÷ 4.9; factorise
$3 - 1 = 2$ s **A1** $t_2 - t_1$

(c) $-24.5 = 19.6t - 4.9t^2 \Rightarrow t^2 - 4t - 5 = 0$ **M1 A1** s = −24.5
$(t - 5)(t + 1) = 0 \Rightarrow t = 5$ s **A1** factorise; t > 0