Exponential Functions and Their Graphs

Specification: AQA 7357: F1, B1, B9 · Edexcel 9MA0: 6.1, 2.1, 2.9

INDEPENDENT

You do

A · Values and shape No calculator.

1 Copy and complete the table for $y = 3^x$. Click a cell to check it.

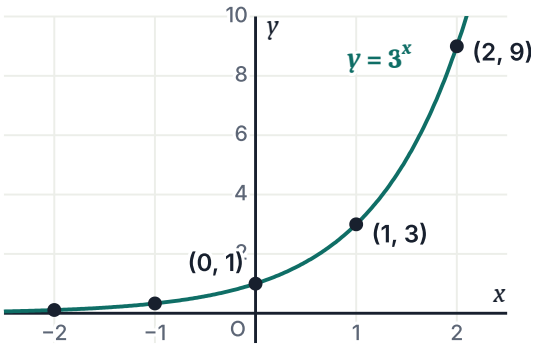
x	−2	−1	0	1	2
y	$\frac{1}{9}$	$\frac{1}{3}$	1	3	9

$3^{-2} = \frac{1}{3^2} = \frac{1}{9}$ — negative power: reciprocal

$3^{-1} = \frac{1}{3}$ — negative power: reciprocal

$3^0 = 1$ — $a^0 = 1$

$3^1 = 3, 3^2 = 9$ — × 3 each step



× 3 each step

2 Write each in the form a^{-x}, where $a > 1$.

(a) $(\frac{1}{6})^x$

(b) 0.25^x

(c) $(\frac{2}{3})^x$

(a) $(\frac{1}{6})^x = (6^{-1})^x = 6^{-x}$ — 1/a = a^{-1}

(b) $0.25 = \frac{1}{4} \Rightarrow 0.25^x = 4^{-x}$ — write 0.25 as ¼

(c) $\frac{2}{3} = \frac{1}{1.5} \Rightarrow (\frac{2}{3})^x = 1.5^{-x}$ — or $(\frac{3}{2})^{-x}$

3 Increasing or decreasing?

(a) 1.01^x

(b) 0.99^x

(c) 2^{-x}

(d) $(\frac{1}{3})^{-x}$

(a) increasing — 1.01 > 1

(b) decreasing — 0 < 0.99 < 1

(c) decreasing — $2^{-x} = (\frac{1}{2})^x$

(d) increasing — $(\frac{1}{3})^{-x} = 3^x$

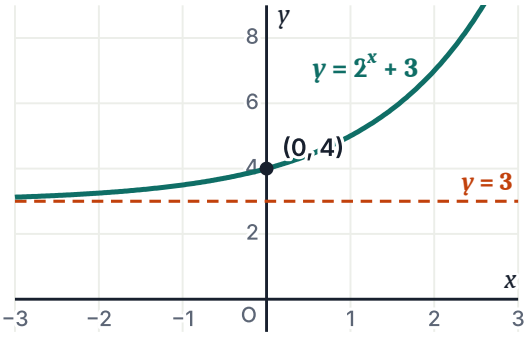
B · Transformations State the y-intercept and the equation of the asymptote, then sketch.

4 Sketch $y = 2^x + 3$.

$f(x) + 3$: translation $\begin{pmatrix}0\\3\end{pmatrix}$, up 3 — f(x) + a: up a

$x = 0$: $y = 2^0 + 3 = 1 + 3 = 4$, so the y-intercept is $(0, 4)$ — y-intercept: put x = 0

Asymptote $y = 3$ — asymptote moves too



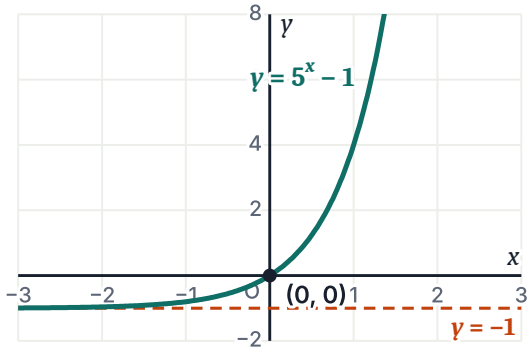
(0, 4); asymptote y = 3

5 Sketch $y = 5^x - 1$.

$f(x) - 1$: translation $\begin{pmatrix}0\\-1\end{pmatrix}$, down 1 — f(x) − a: down a

$x = 0$: $y = 5^0 - 1 = 1 - 1 = 0$, so the y-intercept is $(0, 0)$ — y-intercept: put x = 0

Asymptote $y = -1$ — asymptote moves too



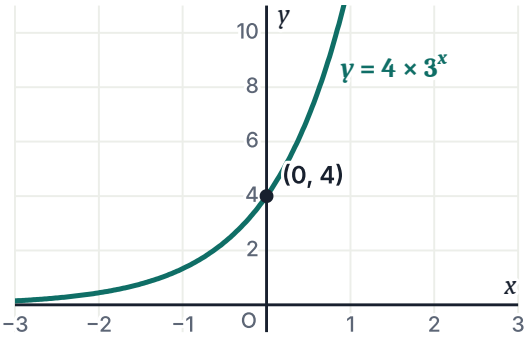
(0, 0); asymptote y = −1

6 Sketch $y = 4 \times 3^x$.

Stretch parallel to the y-axis, scale factor 4 — 4f(x): stretch, s.f. 4

$x = 0$: $y = 4 \times 3^0 = 4 \times 1 = 4$, so the y-intercept is $(0, 4)$ — y-intercept: put x = 0

Asymptote $y = 0$ — asymptote unchanged



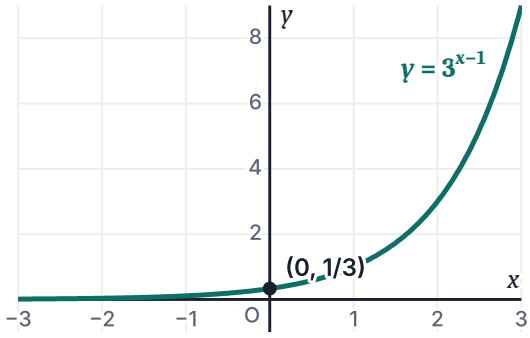
(0, 4); asymptote y = 0

7 Sketch $y = 3^{x-1}$.

$f(x - 1)$: translation $\begin{pmatrix}1\\0\end{pmatrix}$, right 1 — f(x − a): opposite way

$x = 0$: $y = 3^{-1} = \frac{1}{3}$, so the y-intercept is $(0, \frac{1}{3})$ — y-intercept: put x = 0

Asymptote $y = 0$ — asymptote unchanged



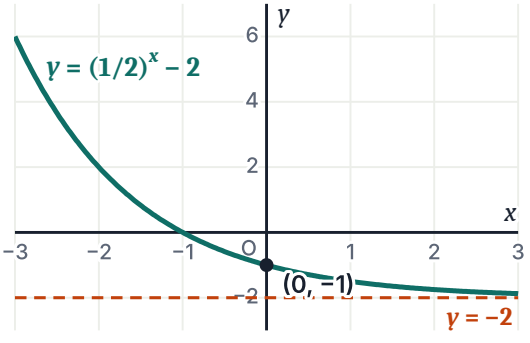
(0, 1/3); asymptote y = 0

8 Sketch $y = (\frac{1}{2})^x - 2$.

Decreasing curve, then $f(x) - 2$: down 2 — 0 < a < 1; down 2

$x = 0$: $y = (\frac{1}{2})^0 - 2 = 1 - 2 = -1$, so the y-intercept is $(0, -1)$ — y-intercept: put x = 0

Asymptote $y = -2$ — asymptote moves too



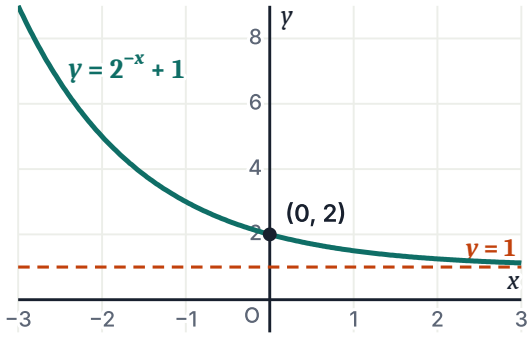
(0, −1); asymptote y = −2

9 Sketch $y = 2^{-x} + 1$.

Reflect $y = 2^x$ in the y-axis, then up 1 — f(−x) + 1: reflect, up 1

$x = 0$: $y = 2^0 + 1 = 1 + 1 = 2$, so the y-intercept is $(0, 2)$ — y-intercept: put x = 0

Asymptote $y = 1$ — asymptote moves too



(0, 2); asymptote y = 1

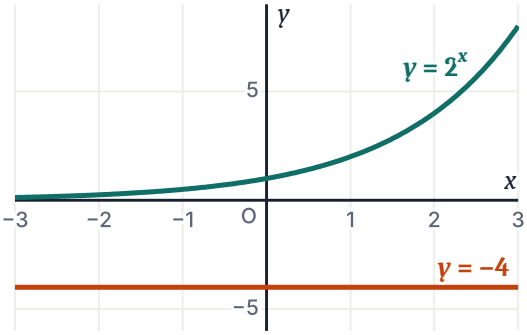
C · Reasoning No calculator.

10 Explain why the equation $2^x = -4$ has no solutions.

$2^x > 0$ for every value of x. $a^x > 0$ always

The graph of $y = 2^x$ never goes below the x-axis, so it never meets $y = -4$.

no intersection



never meets y = −4

11 (a) Show that $3^{x+2} = 9 \times 3^x$.

(b) Describe the graph of $y = 3^{x+2}$ as a transformation of $y = 3^x$ in two different ways.

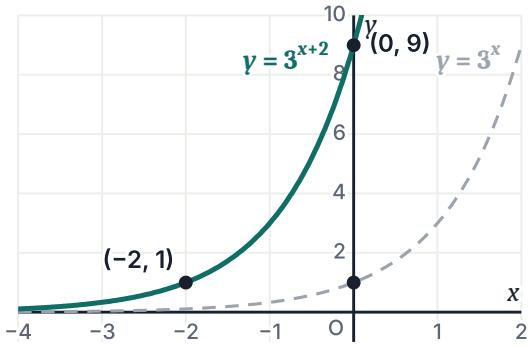
(a) $3^{x+2} = 3^x \times 3^2 = 9 \times 3^x$ $a^{m+n} = a^m a^n$

(b) $f(x+2)$: translation by $\begin{pmatrix} -2 \\ 0 \end{pmatrix}$ (left 2)

f(x + a): opposite way

or $9f(x)$: stretch parallel to the y-axis, scale factor 9

9 × f(x): stretch



left 2 = stretch × 9

12 True or false? Justify each answer.

(a) For every $a > 0$, $y = a^x$ passes through $(1, a)$.

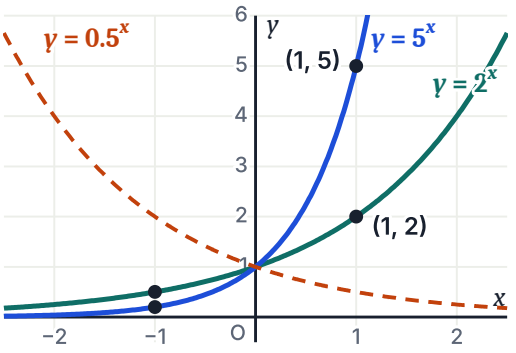
(b) When $x < 0$, $5^x < 2^x$.

(c) $y = 0.5^x$ is an increasing function.

(a) True: $a^1 = a$ substitute x = 1

(b) True: e.g. $x = -1$: $\frac{1}{5} < \frac{1}{2}$ try x = −1

(c) False: $0 < 0.5 < 1$, so it decreases 0 < a < 1: decreasing



x < 0: 5^x below 2^x

13 The curve $y = ka^x$ (with $k, a > 0$) passes through $(1, 12)$ and $(3, 3)$. Find k and a. Is the function increasing or decreasing?

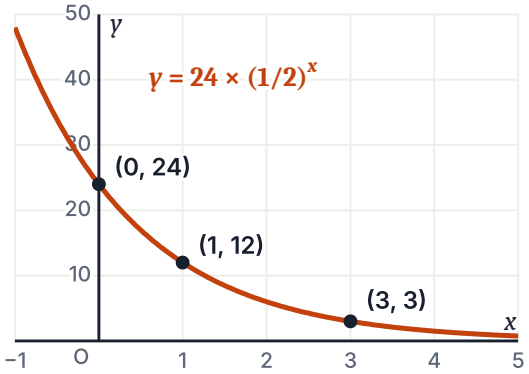
$ka = 12, ka^3 = 3$ — substitute both points

$\frac{ka^3}{ka} = a^2 = \frac{3}{12} = \frac{1}{4}$ — divide: k cancels

$a = \frac{1}{2}$ — a > 0: square root

$\frac{1}{2}k = 12 \Rightarrow k = 24$ — substitute back

Decreasing, since $0 < a < 1$ — 0 < a < 1: decreasing



decreasing: a = ½

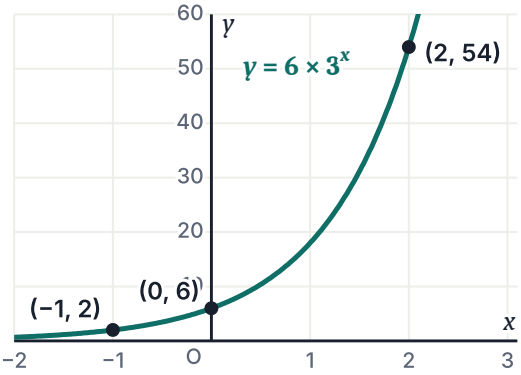
14 The curve $y = ka^x$ (with $k, a > 0$) passes through $(-1, 2)$ and $(2, 54)$. Find k and a.

$ka^{-1} = 2, ka^2 = 54$ — substitute both points

$\frac{ka^2}{ka^{-1}} = a^3 = \frac{54}{2} = 27$ — 2 − (−1) = 3

$a = 3$ — cube root

$\frac{k}{3} = 2 \Rightarrow k = 6$ — substitute back



through (−1, 2) and (2, 54)

15 (a) Show that $8^x = 2^{3x}$.
(b) Hence find the coordinates of the point where $y = 8^x$ meets $y = 2^{x+6}$.

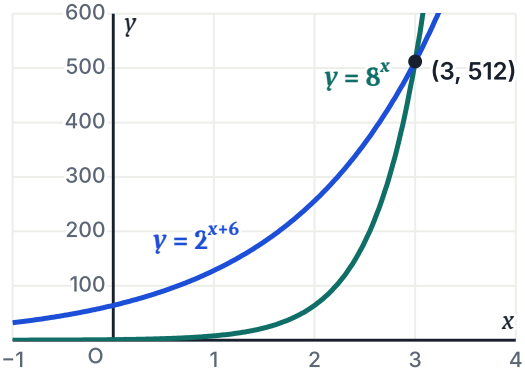
(a) $8^x = (2^3)^x = 2^{3x}$ — $8 = 2^3$; $(a^m)^n = a^{mn}$

(b) $2^{3x} = 2^{x+6} \Rightarrow 3x = x + 6$ — same base: equate powers

$2x = 6 \Rightarrow x = 3$ — solve

$y = 8^3 = 512$ — check: $2^9 = 512$

$(3, 512)$ — the point



they meet at (3, 512)

E · Exam-style question AQA-style. Use the mark scheme to self-assess.

16 The curve C has equation $y = ka^x$, where k and a are positive constants. C passes through the points $(2, 36)$ and $(5, 972)$.

(a) Find the values of a and k. [4 marks]

(b) Sketch C. State the coordinates of the point where C crosses the y-axis and the equation of its asymptote. [2 marks]

(c) Find the value of x for which $y = 324$. [2 marks]

(a) $ka^2 = 36, ka^5 = 972$ **M1** — substitute both points

$a^3 = \dfrac{972}{36} = 27$ **M1** — divide to eliminate k

$a = 3$ **A1** — cube root

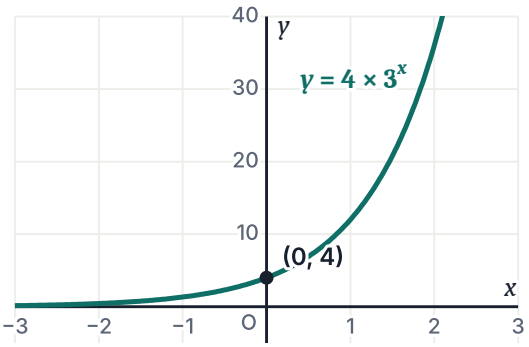
$9k = 36 \Rightarrow k = 4$ **A1** — substitute back

(b) $y = 4 \times 3^x$: an increasing curve, above the x-axis **B1** — shape: k > 0, a > 1

$x = 0$: $y = 4$, so $(0, 4)$; asymptote $y = 0$ **B1** — y-intercept; asymptote

(c) $4 \times 3^x = 324 \Rightarrow 3^x = 81$ **M1** — ÷ 4

$x = 4$ **A1** — $81 = 3^4$



(0, 4); asymptote y = 0