The Exponential Function e^x

Specification: AQA 7357: F1, F2, B9 · Edexcel 9MA0: 6.1, 6.2, 2.9

INDEPENDENT **You do**

A · Differentiate No calculator.

1 Differentiate with respect to x:
(a) e^{8x}
(b) e^{-3x}
(c) $e^{x/4}$
(d) $e^{0.5x}$

(a) $8e^{8x}$ — k = 8

(b) $-3e^{-3x}$ — k = −3

(c) $\frac{1}{4}e^{x/4}$ — k = ¼

(d) $0.5e^{0.5x}$ — k = 0.5

2 Differentiate with respect to x:
(a) $6e^{2x}$
(b) $-e^{5x}$
(c) $10e^{-0.2x}$
(d) $\frac{1}{2}e^{4x}$

(a) $6 \times 2e^{2x} = 12e^{2x}$ — × k = 2

(b) $-1 \times 5e^{5x} = -5e^{5x}$ — × k = 5

(c) $10 \times (-0.2)e^{-0.2x} = -2e^{-0.2x}$ — × k = −0.2

(d) $\frac{1}{2} \times 4e^{4x} = 2e^{4x}$ — × k = 4

3 Differentiate with respect to x:
(a) $e^{2x} + x^3$
(b) $3e^x - 4e^{-x}$
(c) $(e^x + 1)^2$
(d) $e^x(e^{2x} + 3)$

(a) $2e^{2x} + 3x^2$ — term by term

(b) $3e^x + 4e^{-x}$ — −4 × −1 = +4

(c) $e^{2x} + 2e^x + 1 \Rightarrow 2e^{2x} + 2e^x$ — expand first

(d) $e^{3x} + 3e^x \Rightarrow 3e^{3x} + 3e^x$ — expand first

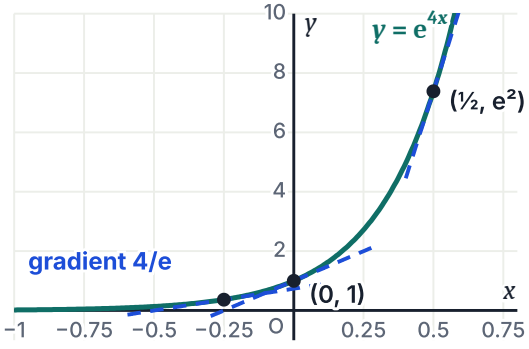
4 Find the exact gradient of $y = e^{4x}$ when
(a) $x = 0$
(b) $x = \frac{1}{2}$
(c) $x = -\frac{1}{4}$

$\frac{dy}{dx} = 4e^{4x}$ — differentiate: × 4

(a) $4e^0 = 4$ — $e^0 = 1$

(b) $4e^2$ — substitute x = ½

(c) $4e^{-1} = \frac{4}{e}$ — $e^{-1} = 1/e$



gradients 4, $4e^2$, $4/e$

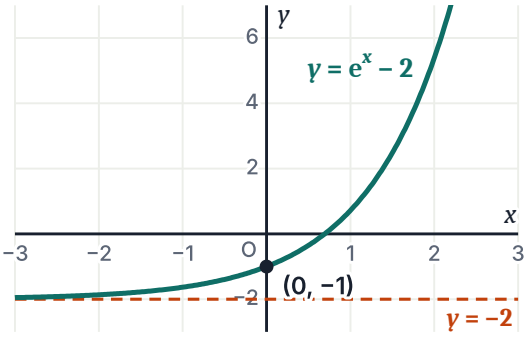
B · Sketch State the y-intercept and the equation of the asymptote, then sketch.

5 Sketch $y = e^x - 2$.

$f(x) - 2$: translation $\begin{pmatrix} 0 \\ -2 \end{pmatrix}$, down 2 — f(x) − a: down a

$x = 0$: $y = e^0 - 2 = 1 - 2 = -1$, so the y-intercept is $(0, -1)$ — y-intercept: put x = 0

Asymptote $y = -2$ — asymptote moves too



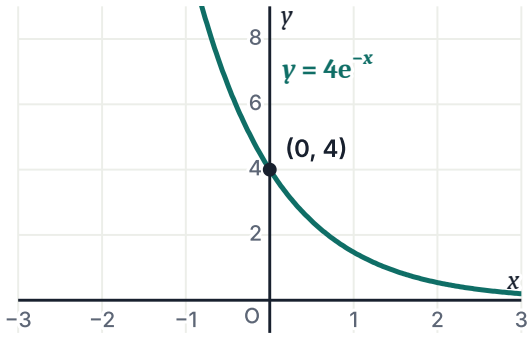
(0, −1); asymptote y = −2

6 Sketch $y = 4e^{-x}$.

Reflect $y = e^x$ in the y-axis, then stretch parallel to the y-axis, scale factor 4 — f(−x), then 4f(x)

$x = 0$: $y = 4e^0 = 4$, so the y-intercept is $(0, 4)$ — y-intercept: put x = 0

Asymptote $y = 0$ — asymptote unchanged



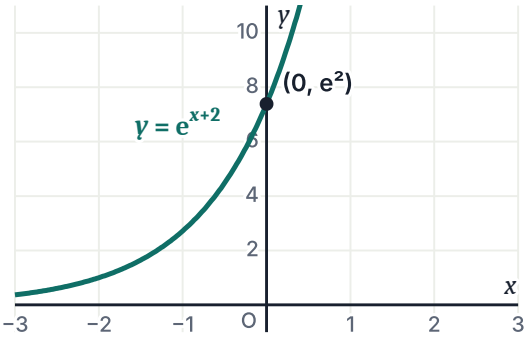
(0, 4); asymptote y = 0

7 Sketch $y = e^{x+2}$.

$f(x + 2)$: translation $\begin{pmatrix} -2 \\ 0 \end{pmatrix}$, left 2 — f(x + a): opposite way

$x = 0$: $y = e^2 \approx 7.39$, so the y-intercept is $(0, e^2)$ — y-intercept: put x = 0

Asymptote $y = 0$ — asymptote unchanged



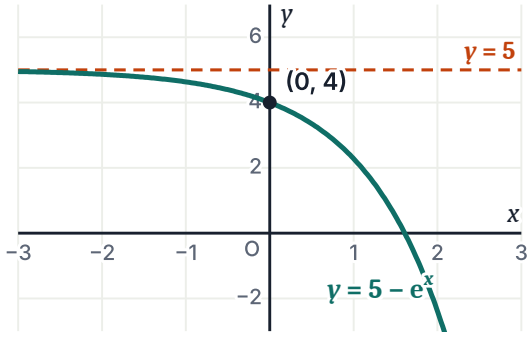
(0, e²); asymptote y = 0

8 Sketch $y = 5 - e^x$.

Reflect $y = e^x$ in the x-axis, then up 5 — −f(x) + 5

$x = 0$: $y = 5 - e^0 = 5 - 1 = 4$, so the y-intercept is $(0, 4)$ — y-intercept: put x = 0

Asymptote $y = 5$ — asymptote moves too



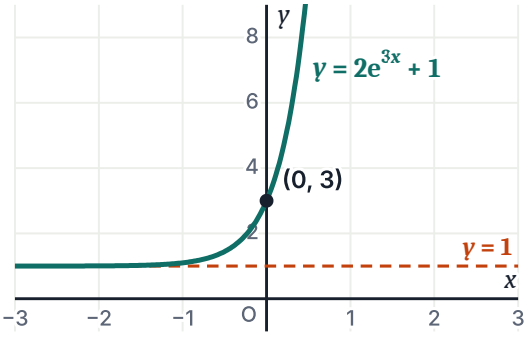
(0, 4); asymptote y = 5

9 Sketch $y = 2e^{3x} + 1$.

Stretch: ⅓ along x, × 2 along y; then up 1 — f(3x), 2f(x), then + 1

$x = 0$: $y = 2e^0 + 1 = 2 + 1 = 3$, so the y-intercept is $(0, 3)$ — y-intercept: put x = 0

Asymptote $y = 1$ — asymptote moves too



(0, 3); asymptote y = 1

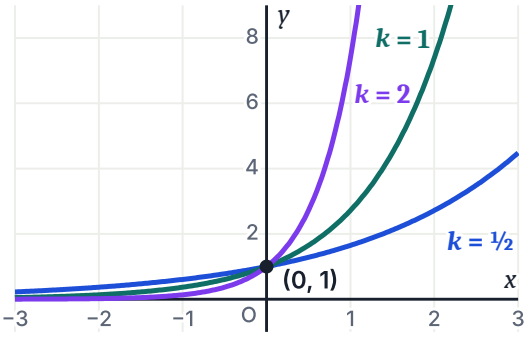
C · Reasoning No calculator.

10 Explain why $y = e^{kx}$ is increasing for every $k > 0$.

$\frac{dy}{dx} = ke^{kx}$ — differentiate: × k

$e^{kx} > 0$ always and $k > 0$, so $\frac{dy}{dx} > 0$ for every x.

positive × positive



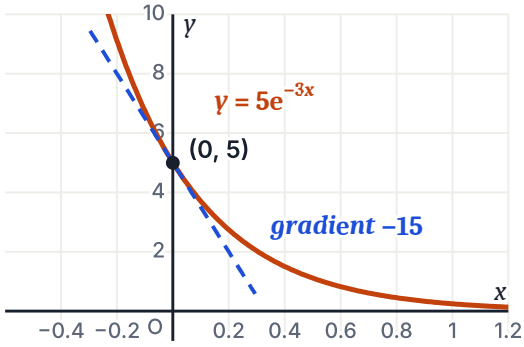
k > 0: every curve increases

11 The curve $y = Ae^{kx}$ passes through $(0, 5)$, and its gradient there is -15. Find A and k.

$x = 0$: $Ae^0 = A = 5$ — y-intercept: put x = 0

$\frac{dy}{dx} = Ake^{kx}$; $x = 0$: $Ak = -15$ — gradient at x = 0

$5k = -15 \Rightarrow k = -3$ — ÷ 5



through (0, 5) with gradient −15

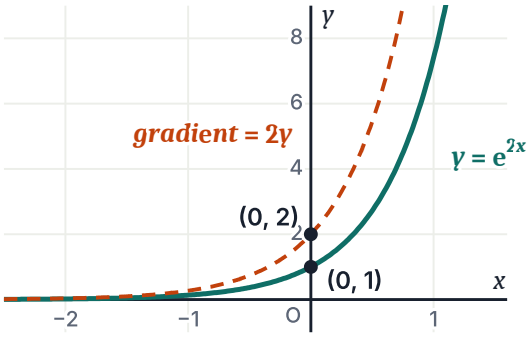
12 (a) Show that for $y = e^{2x}$, the gradient is always twice the y-value.
(b) A population grows at a rate proportional to its size. Explain why a model of the form $P = Ae^{kt}$ is suitable.

(a) $\frac{dy}{dx} = 2e^{2x} = 2y$ — differentiate: × 2

(b) For $P = Ae^{kt}$: $\frac{dP}{dt} = Ake^{kt} = kP$ — differentiate: × k

The rate of change is proportional to P, which is exactly what the population does.

rate ∝ size



(a) the gradient curve is twice the curve

13 Show that the tangent to $y = 3e^{x/2}$ at $x = 2$ passes through the origin.

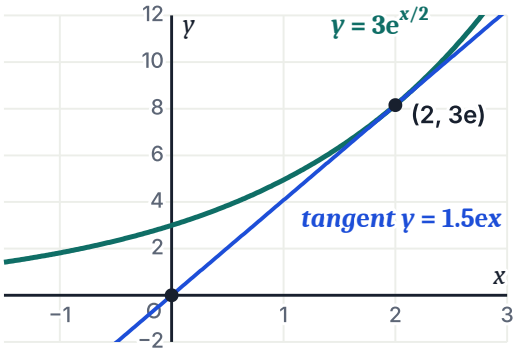
$x = 2$: $y = 3e^1 = 3e$ — point (2, 3e)

$\frac{dy}{dx} = \frac{3}{2}e^{x/2}$; $x = 2$: $m = \frac{3}{2}e$ — gradient at x = 2

$y - 3e = \frac{3}{2}e(x - 2)$ — $y - y_1 = m(x - x_1)$

$y = \frac{3}{2}ex - 3e + 3e = \frac{3}{2}ex$ — expand the brackets

No constant term, so it passes through $(0, 0)$. — no constant: through O



the tangent at (2, 3e) passes through O

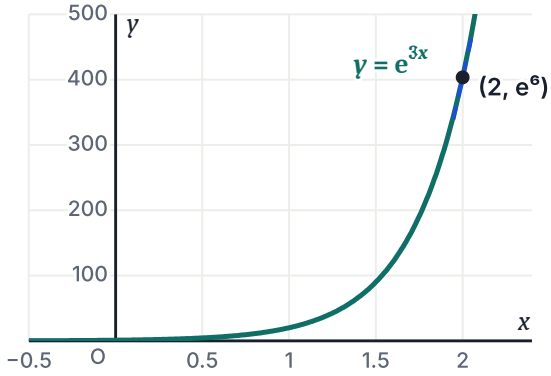
14 Find the coordinates of the point on $y = e^{3x}$ where the gradient is $3e^6$.

$\frac{dy}{dx} = 3e^{3x}$ — differentiate: × 3

$3e^{3x} = 3e^6 \Rightarrow 3x = 6$ — same base: equate powers

$x = 2, y = e^6$ — substitute x = 2

$(2, e^6)$ — the point



gradient $3e^6$ at $(2, e^6)$

15 (a) Show that $y = e^x - x$ has a stationary point at $(0, 1)$.
(b) Determine its nature.

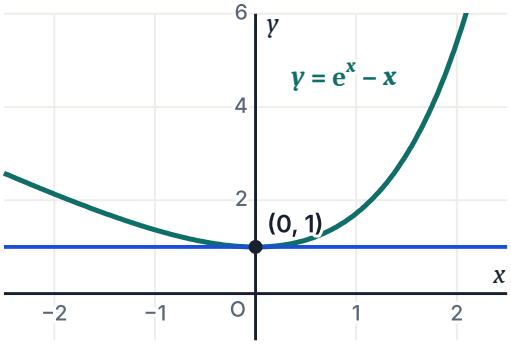
(a) $\frac{dy}{dx} = e^x - 1$ — differentiate

$e^x - 1 = 0 \Rightarrow e^x = 1 \Rightarrow x = 0$ — stationary: dy/dx = 0

$x = 0$: $y = 1 - 0 = 1$, so $(0, 1)$ — substitute x = 0

(b) $\frac{d^2y}{dx^2} = e^x$ — differentiate again

$x = 0$: $\frac{d^2y}{dx^2} = 1 > 0$, so a **minimum** — > 0: minimum



minimum at (0, 1)

E · Exam-style question AQA-style. Use the mark scheme to self-assess.

16 A curve has equation $y = 2e^{2x} - 8x$.

(a) Find $\frac{dy}{dx}$. [2 marks]

(b) Find the equation of the tangent to the curve at the point where $x = 0$. [3 marks]

(c) Find where this tangent crosses the x-axis. [1 mark]

(d) Show that $\frac{d^2y}{dx^2} > 0$ for all x. [2 marks]

(a) $\frac{dy}{dx} = 4e^{2x} - 8$ **M1 A1** — one mark per term

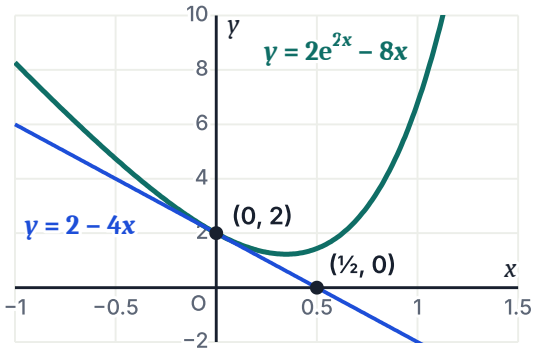
(b) $x = 0$: $y = 2$, $m = 4 - 8 = -4$ **B1 M1** — point and gradient at x = 0

$y - 2 = -4(x - 0) \Rightarrow y = 2 - 4x$ **A1** — $y - y_1 = m(x - x_1)$

(c) $0 = 2 - 4x \Rightarrow x = ½$, so $(½, 0)$ **B1** — x-intercept: put y = 0

(d) $\frac{d^2y}{dx^2} = 8e^{2x}$ **M1** — differentiate again

$e^{2x} > 0$ for all x, so $8e^{2x} > 0$ **A1** — $e^{kx} > 0$ always



(b) tangent y = 2 − 4x (c) crosses at (½, 0)