Exponential Modelling

Specification: AQA 7357: F2, F7 · Edexcel 9MA0: 6.2, 6.7

INDEPENDENT **You do**

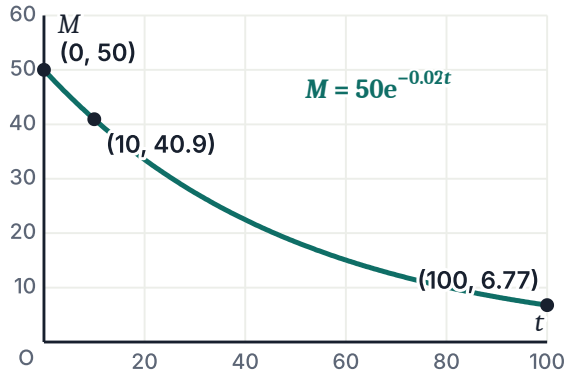
A · Using models

1 A model is $M = 50e^{-0.02t}$. Find M when
(a) $t = 0$
(b) $t = 10$
(c) $t = 100$

(a) $M = 50e^{0} = 50$ — initial value: t = 0

(b) $M = 50e^{-0.02 \times 10} = 50e^{-0.2}$ — substitute t = 10
$= 40.9$ — calculator

(c) $M = 50e^{-0.02 \times 100} = 50e^{-2}$ — substitute t = 100
$= 6.77$ — calculator



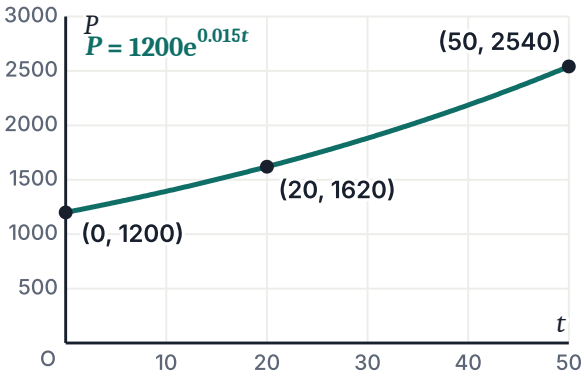
decay towards 0

2 A population, in thousands, is modelled by $P = 1200e^{0.015t}$. Find
(a) the initial population
(b) the population when $t = 20$
(c) the population when $t = 50$

(a) $t = 0$: $P = 1200$ — initial value: t = 0
1200 thousand = 1 200 000 people — units: thousands

(b) $P = 1200e^{0.015 \times 20} = 1200e^{0.3}$ — substitute t = 20
$= 1619.8$ thousand ≈ 1 620 000 — calculator; units

(c) $P = 1200e^{0.015 \times 50} = 1200e^{0.75}$ — substitute t = 50
$= 2540.4$ thousand ≈ 2 540 000 — calculator; units



P in thousands

3 For each model, state whether it shows growth or decay, and give the initial value.
(a) $75e^{0.4t}$
(b) $9e^{-1.2t}$
(c) $300 - 50e^{-t}$
(d) $40 + 10e^{0.1t}$

(a) $k = 0.4 > 0$: growth. $t = 0$: 75 — sign of k; t = 0

(b) $k = -1.2 < 0$: decay. $t = 0$: 9 — sign of k; t = 0

(c) e^{-t} falls, so $300 - 50e^{-t}$ rises towards 300 (growth). — $e^{-t} \to 0$
$t = 0$: $300 - 50 = 250$ — initial value: t = 0

(d) $k = 0.1 > 0$: growth. $t = 0$: $40 + 10 = 50$ — sign of k; t = 0

4 Find the percentage change per unit of time for
(a) $e^{0.03t}$
(b) $e^{-0.4t}$
(c) $e^{0.1t}$

(a) $e^{0.03} = 1.0305$ — multiplier = e^k
$(1.0305 - 1) \times 100$: +3.05% — (multiplier − 1) × 100

(b) $e^{-0.4} = 0.6703$ — multiplier = e^k
$(1 - 0.6703) \times 100$: −33.0% — a decrease

(c) $e^{0.1} = 1.1052$ — multiplier = e^k
$(1.1052 - 1) \times 100$: +10.5% — (multiplier − 1) × 100

5 A model is $N = 60e^{0.5t}$.

(a) Show that $\frac{dN}{dt} = kN$ and state k.

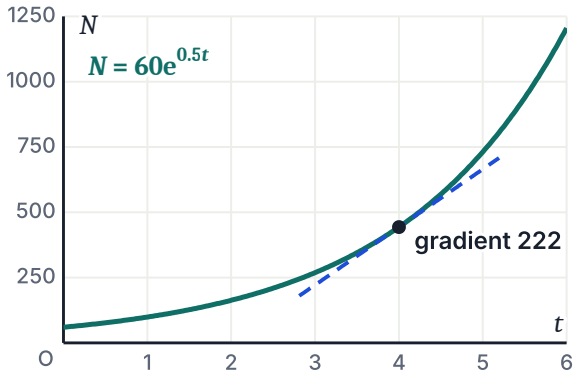
(b) Find the rate of change when $t = 4$.

(a) $\frac{dN}{dt} = 0.5 \times 60e^{0.5t} = 30e^{0.5t}$ — differentiate: × k

$= 0.5 \times 60e^{0.5t} = 0.5N$, so $k = 0.5$ — compare with N

(b) $t = 4$: $30e^{2} = 222$ — substitute t = 4

Increasing at 222 per unit of time — in context



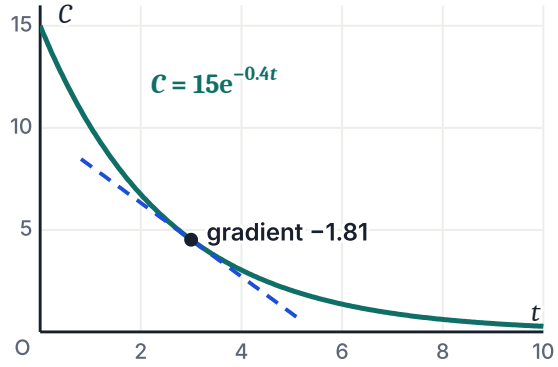
tangent at t = 4

6 The concentration of a medicine is $C = 15e^{-0.4t}$ mg/l, t hours after a dose. Find $\frac{dC}{dt}$ when $t = 3$ and interpret your answer.

$\frac{dC}{dt} = -0.4 \times 15e^{-0.4t} = -6e^{-0.4t}$ — differentiate: × k

$t = 3$: $-6e^{-1.2} = -1.81$ — substitute t = 3

The concentration is falling at 1.81 mg/l per hour. — in context, with units



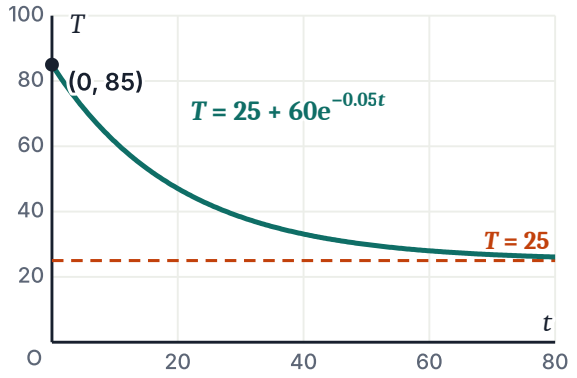
tangent at t = 3: falling

7 A model is $T = 25 + 60e^{-0.05t}$. Show that $\frac{dT}{dt} = -0.05(T - 25)$.

$\frac{dT}{dt} = -0.05 \times 60e^{-0.05t} = -3e^{-0.05t}$ — differentiate: constant → 0

$T - 25 = 60e^{-0.05t}$ — rearrange the model

$-0.05(T - 25) = -3e^{-0.05t} = \frac{dT}{dt}$ — compare: shown



gradient ∝ (T − 25): flattens out

C · Reasoning

8 A population is modelled by $P = 2000e^{0.06t}$. Give two reasons why the model may not be valid for large values of t.

It predicts unlimited growth, but food and space are limited.

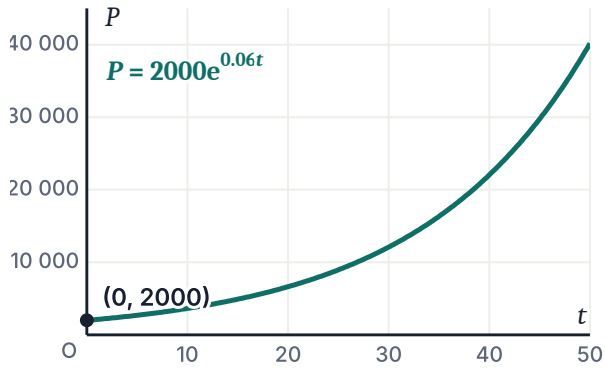
unlimited growth

Births, deaths and migration rates change over time.

conditions change

(Also: disease, predators, the population must be a whole number.)

other limitations



unlimited growth

9 Two models for the value of a laptop, £V, are $V = 1200e^{-0.3t}$ and $V = 1100e^{-0.35t} + 100$.

(a) Show they agree when new.

(b) Compare their values after 10 years.

(c) Which is more realistic in the long term? Why?

(a) $t = 0$: $1200e^0 = 1200$; $1100 + 100 = 1200$ initial value: t = 0

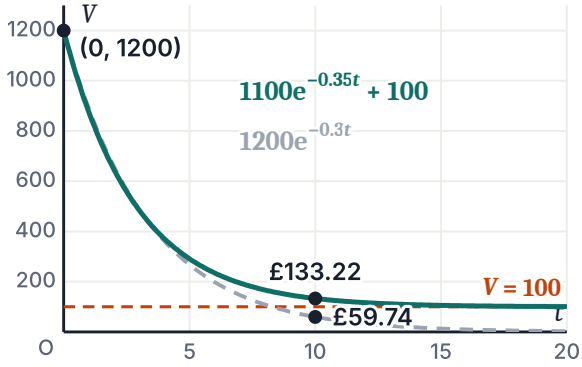
(b) $1200e^{-3} = 59.74$ substitute t = 10

$1100e^{-3.5} + 100 = 133.22$ substitute t = 10

(c) The first tends to £0, the second to £100.

long term: $e^{-kt} \to 0$

The second: a laptop always has some value. in context



second model levels off at £100

10 A radioactive substance is modelled by $N = N_0e^{kt}$.

(a) What does N_0 represent?

(b) Explain why k must be negative.

(a) $t = 0$: $N = N_0e^0 = N_0$ initial value: t = 0

N_0 is the initial amount. in context

(b) The amount decreases, so $\frac{dN}{dt} < 0$. decay: falling

$\frac{dN}{dt} = kN$ and $N > 0$, so $k < 0$. sign of k

D · Algebra

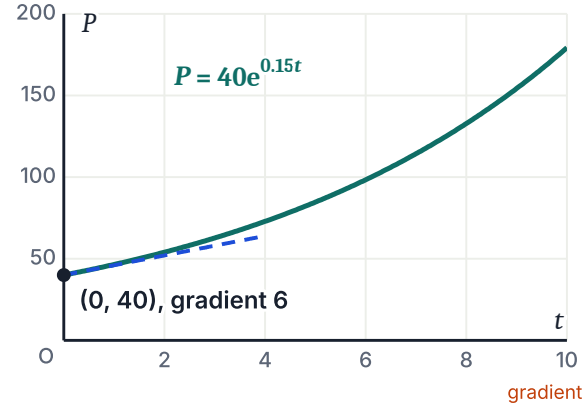
11 A model $P = Ae^{kt}$ has initial value 40, and $\frac{dP}{dt} = 6$ when $t = 0$. Find A and k.

$A = 40$ initial value

$\frac{dP}{dt} = Ake^{kt}$ differentiate: × k

$t = 0$: $40k = 6$ substitute t = 0

$k = 0.15$ ÷ 40



gradient 6 at t = 0

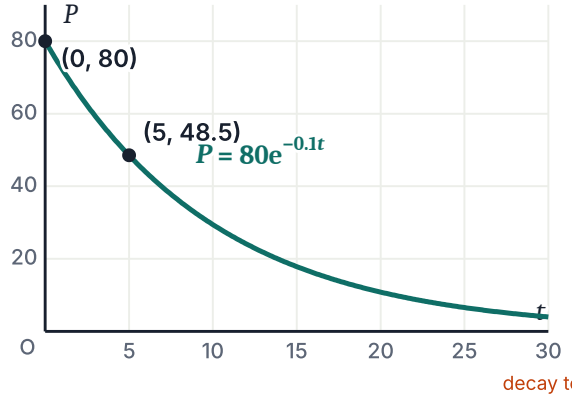
12 A model $P = Ae^{kt}$ has $P = 80$ when $t = 0$, and $\frac{dP}{dt} = -0.1P$. Write down the model and find P when $t = 5$.

$A = 80$ initial value

$k = -0.1$ compare with dP/dt = kP

$P = 80e^{-0.1t}$ write the model

$t = 5$: $80e^{-0.5} = 48.5$ substitute t = 5



decay towards 0

13 Show that $N = 300e^{0.1t}$ can be written as $N = 300 \times b^t$. Find b to 4 d.p. and interpret it.

$e^{0.1t} = (e^{0.1})^t$ — Do Now Q4

$b = e^{0.1} = 1.1052$ — calculator

Each unit of time N is multiplied by 1.1052: 10.5% growth. — interpret the multiplier

E · Exam-style question AQA-style. Use the mark scheme to self-assess.

14 The temperature of a cup of coffee, θ °C, t minutes after it is made is modelled by $\theta = 18 + 72e^{-0.04t}$.

(a) State the initial temperature of the coffee. [1 mark]

(b) Find the temperature after 15 minutes. [2 marks]

(c) Find the rate of change of temperature after 15 minutes and interpret your answer. [3 marks]

(d) State the long-term temperature predicted by the model, and give one reason why the model might need refining. [2 marks]

(a) $t = 0$: $\theta = 18 + 72 = 90$ °C **B1** — initial value: t = 0

(b) $18 + 72e^{-0.6}$ **M1** — substitute t = 15

$= 57.5$ °C **A1** — calculator

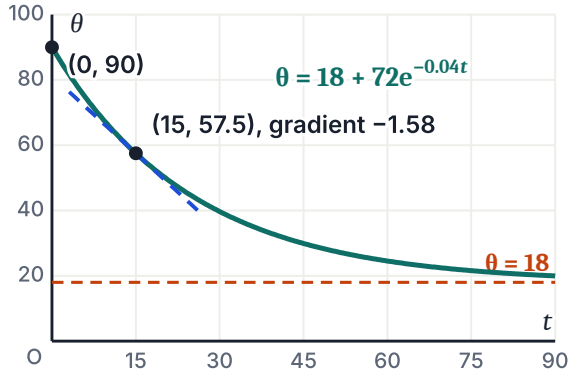
(c) $\frac{d\theta}{dt} = -0.04 \times 72e^{-0.04t} = -2.88e^{-0.04t}$ **M1** — differentiate: × k

$t = 15$: $-2.88e^{-0.6} = -1.58$ **A1** — substitute t = 15

Falling by 1.58 °C per minute **E1** — in context, with units

(d) $e^{-0.04t} \to 0$, so 18 °C **B1** — long term: $e^{-kt} \to 0$

e.g. the room temperature may change; milk may be added **E1** — criticise the model



tangent at t = 15; θ → 18