Exponential Modelling

Specification: AQA 7357: F2, F7 · Edexcel 9MA0: 6.2, 6.7

HOMEWORK Consolidation

1 The value of a car, £V, is modelled by $V = 24000\mathrm{e}^{-0.18t}$, where t is its age in years. Find its value after 3 years, to the nearest pound.
AQA F7 · Edexcel 6.7

2 A country's population, in millions, is modelled by $P = 5.2\mathrm{e}^{0.012t}$, where t is in years. Find the rate of growth when $t = 10$.
AQA F2, F7 · Edexcel 6.2, 6.7

3 A quantity is modelled by $Q = 250\mathrm{e}^{-0.07t}$. By what percentage does it decrease each unit of time?
AQA F7 · Edexcel 6.7

4 A pupil says: "For $A = 30\mathrm{e}^{-0.1t}$, $\frac{dA}{dt} = -3\mathrm{e}^{-0.1t}$, so $k = -3$." Explain the mistake and give the correct value of k.
AQA F2, F7 · Edexcel 6.2, 6.7

5 The temperature of an oven tray is $T = 22 + 63\mathrm{e}^{-0.08t}$ °C, t minutes after it is taken out.
(a) Find its initial temperature.
(b) Find its temperature after 10 minutes.
(c) State its long-term temperature.
AQA F7 · Edexcel 6.7

6 Which grows faster: $P = 500 \times 1.08^t$ or $P = 500\mathrm{e}^{0.08t}$? Explain.
AQA F1, F7 · Edexcel 6.1, 6.7

7 A fish population in a lake is modelled by $F = 1500\mathrm{e}^{0.25t}$, where t is in years. Give two criticisms of the model and suggest one refinement.
AQA F7 · Edexcel 6.7

8 A quantity satisfies $\frac{dN}{dt} = -0.3N$ and $N = 200$ when $t = 0$. Write down a model for N and find N when $t = 4$.
AQA F2, F7 · Edexcel 6.2, 6.7

9 Show that for $y = 40 - 30\mathrm{e}^{-0.5t}$, $\frac{dy}{dt} = 0.5(40 - y)$.
AQA F2, F7 · Edexcel 6.2, 6.7

10 Write $N = 80\mathrm{e}^{0.3t}$ in the form $N = 80 \times b^t$, giving b to 3 d.p. What percentage growth per unit of time does this represent?
AQA F7, B1 · Edexcel 6.7, 2.1

11 *Edexcel-style*. The mass, M grams, of a radioactive substance t years after it was formed is modelled by $M = 60\mathrm{e}^{-0.0004t}$.
(a) State the initial mass. [1 mark]
(b) Find the mass after 1000 years. [2 marks]
(c) Show that $\frac{dM}{dt} = kM$, stating the value of k. [2 marks]
(d) Interpret the sign of k. [1 mark]
(e) Find the percentage of the mass lost each year. [2 marks]
AQA F2, F7 · Edexcel 6.2, 6.7 · Edexcel-style

12 *AQA-style*. The population of a town, P thousand, t years after 2020 is modelled in two ways:
Model A: $P = 40\mathrm{e}^{0.03t}$
Model B: $P = 60 - 20\mathrm{e}^{-0.05t}$
(a) Show that both models give the same population in 2020. [1 mark]
(b) Use each model to predict the population in 2040. [2 marks]
(c) Which model is more suitable for long-term predictions? Explain. [2 marks]
AQA F7 · Edexcel 6.7 · AQA-style