Exponential Modelling

Specification: AQA 7357: F2, F7 · Edexcel 9MA0: 6.2, 6.7

INDEPENDENT **You do**

A · Using models

1 A model is $M = 50e^{-0.02t}$. Find M when
(a) $t = 0$
(b) $t = 10$
(c) $t = 100$

2 A population, in thousands, is modelled by $P = 1200e^{0.015t}$. Find
(a) the initial population
(b) the population when $t = 20$
(c) the population when $t = 50$

3 For each model, state whether it shows growth or decay, and give the initial value.
(a) $75e^{0.4t}$
(b) $9e^{-1.2t}$
(c) $300 - 50e^{-t}$
(d) $40 + 10e^{0.1t}$

4 Find the percentage change per unit of time for
(a) $e^{0.03t}$
(b) $e^{-0.4t}$
(c) $e^{0.1t}$

B · Rates of change

5 A model is $N = 60e^{0.5t}$.
(a) Show that $\frac{dN}{dt} = kN$ and state k.
(b) Find the rate of change when $t = 4$.

6 The concentration of a medicine is $C = 15e^{-0.4t}$ mg/l, t hours after a dose. Find $\frac{dC}{dt}$ when $t = 3$ and interpret your answer.

7 A model is $T = 25 + 60e^{-0.05t}$. Show that $\frac{dT}{dt} = -0.05(T - 25)$.

C · Reasoning

8 A population is modelled by $P = 2000e^{0.06t}$. Give two reasons why the model may not be valid for large values of t.

9 Two models for the value of a laptop, £V, are $V = 1200e^{-0.3t}$ and $V = 1100e^{-0.35t} + 100$.
(a) Show they agree when new.
(b) Compare their values after 10 years.
(c) Which is more realistic in the long term? Why?

10 A radioactive substance is modelled by $N = N_0e^{kt}$.
(a) What does N_0 represent?
(b) Explain why k must be negative.

D · Algebra

11 A model $P = Ae^{kt}$ has initial value 40, and $\frac{dP}{dt} = 6$ when $t = 0$. Find A and k.

12 A model $P = Ae^{kt}$ has $P = 80$ when $t = 0$, and $\frac{dP}{dt} = -0.1P$. Write down the model and find P when $t = 5$.

13 Show that $N = 300e^{0.1t}$ can be written as $N = 300 \times b^t$. Find b to 4 d.p. and interpret it.

E · Exam-style question

AQA-style. Use the mark scheme to self-assess.

14 The temperature of a cup of coffee, θ °C, t minutes after it is made is modelled by $\theta = 18 + 72e^{-0.04t}$.
(a) State the initial temperature of the coffee. [1 mark]
(b) Find the temperature after 15 minutes. [2 marks]
(c) Find the rate of change of temperature after 15 minutes and interpret your answer. [3 marks]
(d) State the long-term temperature predicted by the model, and give one reason why the model might need refining. [2 marks]