Logarithms

Specification: AQA 7357: F3, B1 · Edexcel 9MA0: 6.3, 2.1

INDEPENDENT

You do

A · Switch and evaluate

1 Write as a logarithm:
(a) $2^6 = 64$
(b) $10^{-1} = 0.1$
(c) $8^{⅓} = 2$
(d) $7^0 = 1$

(a) $\log_2 64 = 6$ — base 2, power 6

(b) $\log_{10} 0.1 = -1$ — base 10, power −1

(c) $\log_8 2 = \frac{1}{3}$ — base 8, power ⅓

(d) $\log_7 1 = 0$ — base 7, power 0

2 Write using a power:
(a) $\log_4 1024 = 5$
(b) $\log_9 3 = \frac{1}{2}$
(c) $\log_2 0.25 = -2$
(d) $\log_a b = c$

(a) $4^5 = 1024$ — rewrite as a power

(b) $9^{½} = 3$ — rewrite as a power

(c) $2^{-2} = 0.25$ — rewrite as a power

(d) $a^c = b$ — base to the answer

3 Find the value of:
(a) $\log_2 128$
(b) $\log_3 243$
(c) $\log_{10} 1000000$
(d) $\log_7 7$

(a) $2^7 = 128$, so 7 — 2 to what power?

(b) $3^5 = 243$, so 5 — 3 to what power?

(c) $10^6 = 1000000$, so 6 — count the zeros

(d) $7^1 = 7$, so 1 — $\log_a a = 1$

4 Find the value of:
(a) $\log_2 \frac{1}{32}$
(b) $\log_5 0.2$
(c) $\log_4 8$
(d) $\log_2 \sqrt{2}$

(a) $2^{-5} = \frac{1}{32}$, so −5 — negative power: reciprocal

(b) $0.2 = \frac{1}{5} = 5^{-1}$, so −1 — 0.2 = 1/5

(c) $4^{3/2} = 2^3 = 8$, so $\frac{3}{2}$ — √4 = 2, then cube

(d) $\sqrt{2} = 2^{½}$, so $\frac{1}{2}$ — power ½: square root

B · Solve

5 Solve:
(a) $\log_2 x = 6$
(b) $\log_5 x = -1$
(c) $\log_9 x = \frac{1}{2}$

(a) $x = 2^6 = 64$ — rewrite as a power

(b) $x = 5^{-1} = \frac{1}{5}$ — negative power: reciprocal

(c) $x = 9^{½} = 3$ — power ½: square root

6 Solve:
(a) $\log_x 125 = 3$
(b) $\log_x 16 = 4$
(c) $\log_x 0.5 = -1$

(a) $x^3 = 125 \Rightarrow x = 5$ — cube root

(b) $x^4 = 16 \Rightarrow x = 2$ — 4th root; base > 0

(c) $x^{-1} = 0.5 \Rightarrow \frac{1}{x} = \frac{1}{2} \Rightarrow x = 2$ — $x^{-1} = 1/x$

7 Solve:
(a) $\log_3 (x - 2) = 2$
(b) $\log_2 (3x + 1) = 4$

(a) $x - 2 = 3^2 = 9$ — rewrite as a power
$x = 11$ — + 2

(b) $3x + 1 = 2^4 = 16$ — rewrite as a power
$3x = 15 \Rightarrow x = 5$ — − 1, then ÷ 3

8 Use a calculator to find, to 3 d.p.:
(a) $\log_2 30$
(b) $\log_7 100$
(c) $\log 0.5$

(a) $\log_2 30 = 4.907$ — calculator: log□ key

(b) $\log_7 100 = 2.367$ — calculator: log□ key

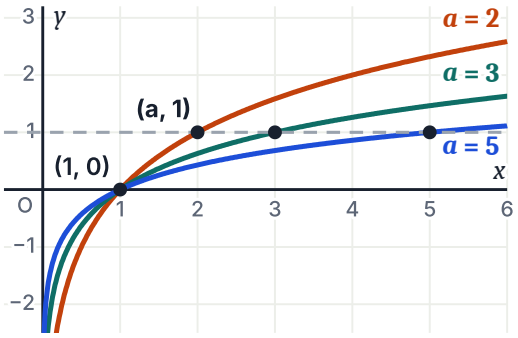
(c) $\log 0.5 = -0.301$ — log is base 10

C · Reasoning No calculator.

9 Explain why $\log_a 1 = 0$ and $\log_a a = 1$ for every $a > 0, a \neq 1$.

$a^0 = 1$ for every a, so $\log_a 1 = 0$ — $a^0 = 1$

$a^1 = a$, so $\log_a a = 1$ — $a^1 = a$

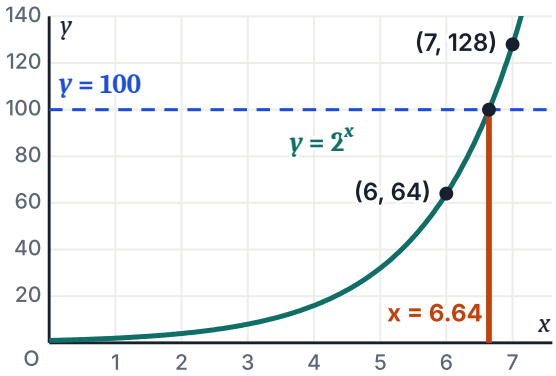


$y = \log_a x$: every curve through (1, 0) and (a, 1)

10 Between which two consecutive integers does $\log_2 100$ lie? Explain.

$2^6 = 64 < 100 < 128 = 2^7$ — powers of 2 either side

Between 6 and 7 — so between 6 and 7



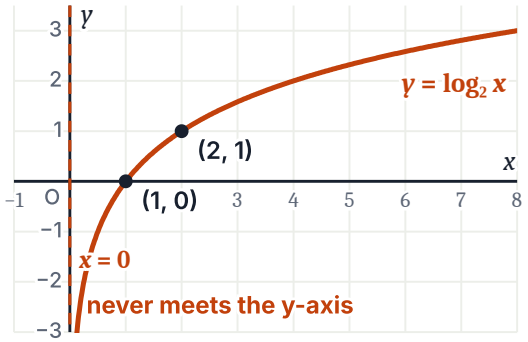
64 < 100 < 128: $2^x = 100$ between x = 6 and 7

11 Explain why the graph of $y = \log_2 x$ has no y-intercept.

On the y-axis, $x = 0$. — y-axis: x = 0

$\log_2 0$ would need $2^y = 0$, which never happens. — $2^y > 0$ always

The y-axis is an asymptote. — asymptote x = 0



the y-axis is an asymptote: no y-intercept

D · Algebra

12 Simplify:
(a) $\log_a a^7$
(b) $a^{\log_a 5}$

(a) a to what power gives a^7? 7 — the log is the power

(b) $\log_a 5$ is the power of a that gives 5, so $a^{\log_a 5} = 5$ — log and power undo

13 Solve $\log_2 (\log_3 x) = 2$.

$\log_3 x = 2^2 = 4$ — outer log first

$x = 3^4 = 81$ — rewrite as a power

14 Solve $\log_4 x = \log_2 8$.

$\log_2 8 = 3$ — evaluate the right side

$\log_4 x = 3 \Rightarrow x = 4^3 = 64$ — rewrite as a power

E · Exam-style question AQA-style. No calculator. Use the mark scheme to self-assess.

15 (a) Write down the value of $\log_5 125$. [1 mark]

(b) Find the value of $\log_4 0.5$. [2 marks]

(c) Solve $\log_x 27 = \frac{3}{2}$. [3 marks]

(a) $5^3 = 125$, so 3 **B1** — 5 to what power?

(b) $0.5 = \frac{1}{2} = 4^{-½}$ **M1** — ½ = 1/√4

$\log_4 0.5 = -\frac{1}{2}$ **A1** — the log is the power

(c) $x^{3/2} = 27$ **M1** — rewrite as a power

$x = 27^{⅔}$ **M1** — raise both sides to ⅔

$x = (\sqrt[3]{27})^2 = 3^2 = 9$ **A1** — cube root, then square