Laws of Logarithms

Specification: AQA 7357: F3, F4 · Edexcel 9MA0: 6.3, 6.4

INDEPENDENT **You do**

A · Single logarithms

1 Write as a single logarithm:
(a) $\log_3 7 + \log_3 4$
(b) $\log_5 36 - \log_5 4$
(c) $2\log_2 5$
(d) $\log_{10} 2 + \log_{10} 50$

(a) $\log_3 (7 \times 4) = \log_3 28$ — multiplication law

(b) $\log_5 (36 \div 4) = \log_5 9$ — division law

(c) $\log_2 5^2= \log_2 25$ — power law

(d) $\log_{10} (2 \times 50) = \log_{10} 100 = 2$ — multiplication law; $10^2 = 100$

2 Write as a single logarithm:
(a) $3\log_2 3 + \log_2 2$
(b) $2\log_7 4 - 3\log_7 2$
(c) $\frac{1}{2}\log_3 49 + \log_3 2$
(d) $\log_5 8 - \frac{1}{3}\log_5 27$

(a) $\log_2 27 + \log_2 2 = \log_2 54$ — power law first; $3^3 = 27$

(b) $\log_7 16 - \log_7 8 = \log_7 2$ — power law first; $16 \div 8$

(c) $\log_3 7 + \log_3 2 = \log_3 14$ — $49^{\frac{1}{2}} = 7$

(d) $\log_5 8 - \log_5 3 = \log_5 \frac{8}{3}$ — $27^{\frac{1}{3}} = 3$

3 Find the value of:
(a) $\log_4 2 + \log_4 32$
(b) $\log_2 24 - \log_2 3$
(c) $2\log_6 3 + \log_6 4$
(d) $\log_{10} 25 + \log_{10} 4$

(a) $\log_4 64 = 3$ — $2 \times 32 = 64 = 4^3$

(b) $\log_2 8 = 3$ — $24 \div 3 = 8 = 2^3$

(c) $\log_6 9 + \log_6 4 = \log_6 36 = 2$ — power law; $6^2 = 36$

(d) $\log_{10} 100 = 2$ — $25 \times 4 = 100 = 10^2$

B · Expanding

4 Write in terms of $\log_a x$, $\log_a y$ and $\log_a z$:
(a) $\log_a (x^4 y)$
(b) $\log_a \left(\frac{x}{y^5}\right)$
(c) $\log_a \left(\frac{\sqrt{x}y^3}{z}\right)$
(d) $\log_a \left(\frac{x}{a^3}\right)$

(a) $\log_a x^4 + \log_a y = 4\log_a x + \log_a y$ — multiplication, then power law

(b) $\log_a x - \log_a y^5 = \log_a x - 5\log_a y$ — division, then power law

(c) $\frac{1}{2}\log_a x + 3\log_a y - \log_a z$ — top: add; bottom: subtract

(d) $\log_a x - \log_a a^3 = \log_a x - 3$ — $\log_a a^3 = 3$

5 Given $\log_a 2 = p$ and $\log_a 3 = q$, write in terms of p and q:
(a) $\log_a 6$
(b) $\log_a 18$
(c) $\log_a \frac{8}{9}$
(d) $\log_a 1.5$

(a) $\log_a 2 + \log_a 3 = p + q$ — $6 = 2 \times 3$

(b) $\log_a 2 + 2\log_a 3 = p + 2q$ — $18 = 2 \times 3^2$

(c) $3\log_a 2 - 2\log_a 3 = 3p - 2q$ — $8 = 2^3$, $9 = 3^2$

(d) $\log_a 3 - \log_a 2 = q - p$ — $1.5 = 3 \div 2$

C · Reasoning

6 Prove the division law: $\log_a x - \log_a y = \log_a \frac{x}{y}$.

Let $\log_a x = p, \log_a y = q$ — name the logs

$x = a^p, y = a^q \Rightarrow \frac{x}{y} = a^{p-q}$ — subtract the powers

$\log_a \frac{x}{y} = p - q = \log_a x - \log_a y$ — the log is the power

7 Show that $\log_a \frac{1}{x} = -\log_a x$. Hence find $\log_2 \frac{1}{32} + \log_2 4$.

$\log_a \frac{1}{x} = \log_a x^{-1} = -\log_a x$ — power law, k = −1

$\log_2 \frac{1}{32} = -\log_2 32 = -5$, $\log_2 4 = 2$ — $2^5 = 32$, $2^2 = 4$

$-5 + 2 = -3$ — add

8 A pupil says $\log_2 (x + 4) = \log_2 x + 2$ for all $x > 0$. Show this is false, and find the one value of x for which it is true.

$x = 4$: $\log_2 8 = 3$ but $\log_2 4 + 2 = 4$ — a counter-example

$\log_2 x + 2 = \log_2 x + \log_2 4 = \log_2 4x$ — $2 = \log_2 4$

$x + 4 = 4x \Rightarrow x = \frac{4}{3}$ — equate the insides

D · Solving equations

9 Solve $\log_3 x + \log_3 4 = 2$.

$\log_3 4x = 2$ — multiplication law

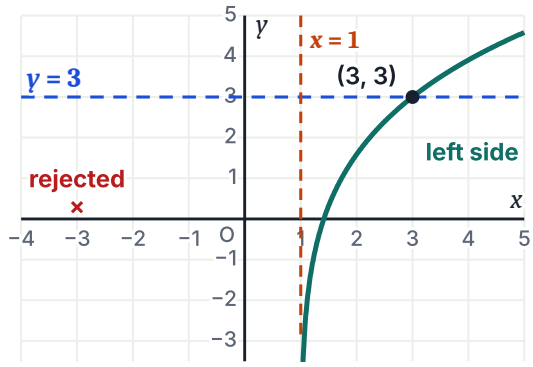
$4x = 3^2 = 9 \Rightarrow x = \frac{9}{4}$ — rewrite as a power

10 Solve $\log_2 (x + 1) + \log_2 (x - 1) = 3$.

$\log_2 (x + 1)(x - 1) = 3$ — multiplication law

$x^2 - 1 = 2^3 = 8 \Rightarrow x^2 = 9$ — rewrite as a power

$x = 3$; reject $x = -3$: $\log_2 (-2)$ is undefined — check every log is defined



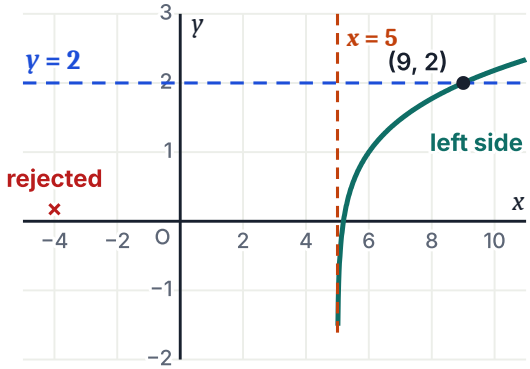
x = 3; x = −3 rejected (needs x > 1)

11 Solve $\log_6 x + \log_6 (x - 5) = 2$.

$x(x - 5) = 6^2 = 36 \Rightarrow x^2 - 5x - 36 = 0$ — one log; rewrite as a power

$(x - 9)(x + 4) = 0$ — factorise

$x = 9$; reject $x = -4$: $\log_6 (-4)$ is undefined — check every log is defined



x = 9; x = −4 rejected (needs x > 5)

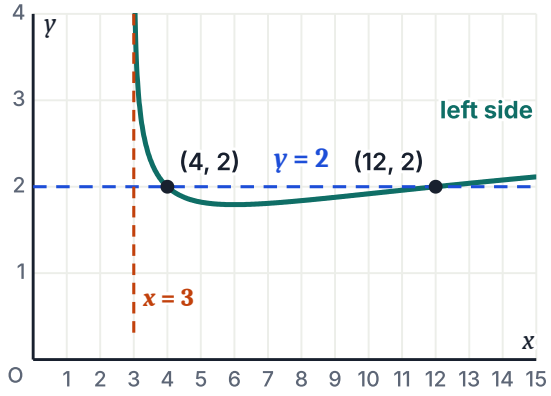
12 Solve $2 \log_4 x - \log_4 (x - 3) = 2$.

$\log_4 x^2 - \log_4 (x - 3) = \log_4 \frac{x^2}{x - 3} = 2$ — power law first; division law

$x^2 = 16(x - 3) \Rightarrow x^2 - 16x + 48 = 0$ — rewrite as a power; × (x − 3)

$(x - 4)(x - 12) = 0$ — factorise

$x = 4$ or $x = 12$: both make the number inside every log positive ($x > 3$) ✓ — check: arguments positive



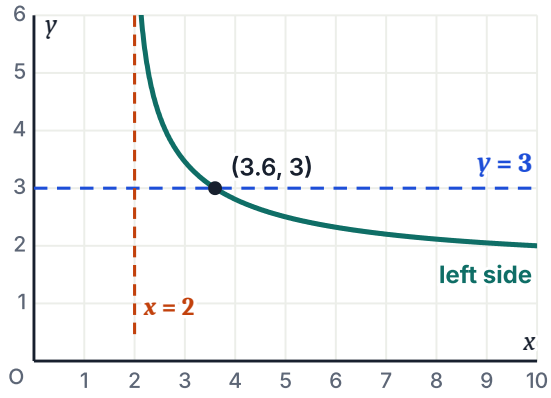
two meeting points: x = 4 and x = 12 (both > 3)

13 Solve $\log_2(3x+2) - \log_2(x-2) = 3$.

$\log_2 \dfrac{3x+2}{x-2} = 3 \Rightarrow 3x+2 = 8(x-2)$ — division law; rewrite as a power

$3x+2 = 8x-16 \Rightarrow 5x = 18 \Rightarrow x = 3.6$ — x terms to one side

Check: $3x+2 = 12.8 > 0$ and $x-2 = 1.6 > 0$ ✓ — check: arguments positive



x = 3.6 (x − 2 = 1.6 > 0)

E · Exam-style question AQA-style. No calculator. Use the mark scheme to self-assess.

14 (a) Show that $\log_2(x^2-9) - \log_2(x-3) = \log_2(x+3)$, for $x > 3$. [2 marks]

(b) Hence solve $\log_2(x^2-9) - \log_2(x-3) = 3$. [2 marks]

(c) Solve $\log_2 x + \log_2(x+6) = 4$. [4 marks]

(a) $\log_2 \dfrac{x^2-9}{x-3}$ M1 — division law

$= \log_2 \dfrac{(x-3)(x+3)}{x-3} = \log_2(x+3)$ A1 — difference of two squares

(b) $\log_2(x+3) = 3 \Rightarrow x+3 = 2^3 = 8$ M1 — rewrite as a power

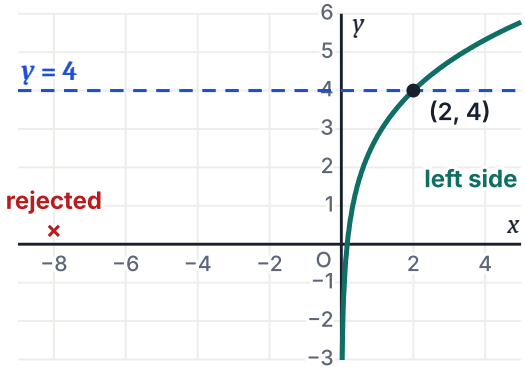
$x = 5$ (and $5 > 3$ ✓) A1 — − 3; check x > 3

(c) $\log_2 x(x+6) = 4$ M1 — multiplication law

$x(x+6) = 2^4 = 16 \Rightarrow x^2+6x-16 = 0$ M1 — rewrite as a power

$(x+8)(x-2) = 0$ A1 — factorise

$x = 2$; reject $x = -8$: $\log_2(-8)$ is undefined A1 — check every log is defined



(c) x = 2; x = −8 rejected (needs x > 0)