Laws of Logarithms

Specification: AQA 7357: F3, F4 · Edexcel 9MA0: 6.3, 6.4

INDEPENDENT **You do**

A · Single logarithms

1 Write as a single logarithm:
(a) $\log_3 7 + \log_3 4$
(b) $\log_5 36 - \log_5 4$
(c) $2\log_2 5$
(d) $\log_{10} 2 + \log_{10} 50$

2 Write as a single logarithm:
(a) $3\log_2 3 + \log_2 2$
(b) $2\log_7 4 - 3\log_7 2$
(c) $\frac{1}{2}\log_3 49 + \log_3 2$
(d) $\log_5 8 - \frac{1}{3}\log_5 27$

3 Find the value of:
(a) $\log_4 2 + \log_4 32$
(b) $\log_2 24 - \log_2 3$
(c) $2\log_6 3 + \log_6 4$
(d) $\log_{10} 25 + \log_{10} 4$

B · Expanding

4 Write in terms of $\log_a x$, $\log_a y$ and $\log_a z$:
(a) $\log_a (x^4 y)$
(b) $\log_a \left(\frac{x}{y^5}\right)$
(c) $\log_a \left(\frac{\sqrt{x}y^3}{z}\right)$
(d) $\log_a \left(\frac{x}{a^3}\right)$

5 Given $\log_a 2 = p$ and $\log_a 3 = q$, write in terms of p and q:
(a) $\log_a 6$
(b) $\log_a 18$
(c) $\log_a \frac{8}{9}$
(d) $\log_a 1.5$

C · Reasoning

6 Prove the division law: $\log_a x - \log_a y = \log_a \frac{x}{y}$.

7 Show that $\log_a \frac{1}{x} = -\log_a x$. Hence find $\log_2 \frac{1}{32} + \log_2 4$.

8 A pupil says $\log_2 (x + 4) = \log_2 x + 2$ for all $x > 0$. Show this is false, and find the one value of x for which it is true.

D · Solving equations

9 Solve $\log_3 x + \log_3 4 = 2$.

10 Solve $\log_2 (x + 1) + \log_2 (x - 1) = 3$.

11 Solve $\log_6 x + \log_6 (x - 5) = 2$.

12 Solve $2\log_4 x - \log_4 (x - 3) = 2$.

13 Solve $\log_2 (3x + 2) - \log_2 (x - 2) = 3$.

E · Exam-style question AQA-style. No calculator. Use the mark scheme to self-assess.

14 (a) Show that $\log_2 (x^2 - 9) - \log_2 (x - 3) = \log_2 (x + 3)$, for $x > 3$. [2 marks]
(b) Hence solve $\log_2 (x^2 - 9) - \log_2 (x - 3) = 3$. [2 marks]
(c) Solve $\log_2 x + \log_2 (x + 6) = 4$. [4 marks]